Re: 7−th power

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- *From*: israel@xxxxxxxxxxx (Robert Israel)
- *Date*: 30 Jun 2006 02:13:19 GMT

In article <1151603590.953912.111410@xxxxxxxxxxxxxxxxxxxxxxxxxxxx>,
Deep <deepkdeb@xxxxxxxxx> wrote:

Please don't top−post.

> Thank you very much for your comments. Perhaps you have overlooked the
> fact that none of a, b, m, n is a pefact square.

I have not.

> Therefore it excludes
> thev case a = 1.

Yes. I wanted solutions of the equation $r^7 = 5/a^3 - 20/a^2 + 16/a$
where a and r are rational and a is not a square. But even without
that restriction, the only rational solution I could find was a = 1.

> Also how did you come to the conclusion that numerator
> and denominator are < 1000?

I didn't. I did a brute−force search for solutions with
numerator and denominator < 1000. Since I didn't find any, other
than the trivial a=1, you can conclude that a nontrivial solution
has numerator or denominator >= 1000.

A bit more can be said. Suppose p is a prime (other than 2 or 5)
such that the p−adic order of a is k <> 0 (i.e. $a = p^k q$ where
the numerator and denominator of the rational number q are
coprime to p). Then the p−adic order of $R = 5/a^3 - 20/a^2 + 16/a$
is −k if k < 0 or −3k if k > 0.
Since the p−adic order of r^7 must be a multiple of 7, that implies
k is divisible by 7.
For p = 2, if the 2−adic order of a is k, then the 2−adic

order of R is −k+4 if k <= −2 and −3k if k > −2. This implies that the 2−adic order of a must be a negative number == 4 mod 7 or a nonnegative number divisible by 7.
Similarly, the 5−adic order of a must be either a nonpositive number divisible by 7 or a positive number == 5 mod 7.

I still don't know any solution other than a=1, and I suspect none exists. Your statement is equivalent to saying there is at most one solution where a is not a square.

Robert Israel israel@xxxxxxxxxxx
Department of Mathematics http://www.math.ubc.ca/~israel
University of British Columbia Vancouver, BC, Canada

> A liile clarifcication from you will be very helpful.
> Greetings.
> Robert Israel wrote:
>
>> In article
>> <1151541469.122083.204670@xxxxxxxxxxxxxxxxxxxxxxxxxxxx>,
>> Deep <deepkdeb@xxxxxxxxx> wrote:
>>
>>> Consider the following two equations under the given conditions.
>>>
>>> A = 5x −20x^3 + 16x^5 (1)
>>
>> A = x (5 − 20 x^2 + 16 x^4)
>>
>>> B = 5y − 20y^3 + 16y^5 (2)
>>>
>>> where x = (a)^1/2 and y = (b)^1/2, a, b are real rational and none of a
>>> or b is a perfect square.
>>
>> A = a^(1/2) (5 − 20 a + 16 a^2)
>>
>>> Statement:
>>>
>>> If A = (M)^7 and B = (N)^7 where M = (m)^1/2 and N = (n)^1/2; m, n are
>>> real, rational and are not perfect squares then m = n which in turn
>>> implies x = y.

> Any comment upon the correctness of the above statement
> will be
> appreciated.

So you want a^(1/2) (5 − 20 a + 16 a^2) = m^(7/2).

(m/a)^(1/2) = (5 − 20 a + 16 a^2)/m^3 is rational. Call it r.
So m = r^2 a and

r^7 = 5/a^3 − 20/a^2 + 16/a.

One solution is a=1, r=1, m=1. I don't know if that is the only rational solution. If it is, your statement is vacuously true.
Unless I've made a mistake, there are no other rational solutions where the absolute value of the numerator and denominator of a are both less than 1000.

Robert Israel israel@xxxxxxxxxxx
Department of Mathematics http://www.math.ubc.ca/~israel
University of British Columbia Vancouver, BC, Canada

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