Re: Is magnitude more fundamental than the real numbers?

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- *From*: "Gene Ward Smith" <genewardsmith@xxxxxxxxx>
- *Date*: 30 Jun 2006 18:00:02 −0700

Timothy Golden BandTechnology.com wrote:

> Magnitudes are not real numbers. They are much simpler than real
> numbers.
>
> from http://en.wikipedia.org/wiki/Magnitude_%28mathematics%29 :
>
> The magnitude of a mathematical object is its size:
> a property by which it can be larger or smaller than other objects
> of the same kind; in technical terms, an ordering of the class of
> objects
> to which it belongs.

Which doesn't contradict anything I said.

More quotes from the same article:

The magnitude of a real number is usually called the absolute value or modulus. It is written | x |, and is defined by:

| x | = x, if x = 0
| x | = −x, if x < 0

and

Similarly, the magnitude of a complex number, called the modulus, gives the distance from zero in the Argand diagram. The formula for the modulus is the same as that for Pythagoras' theorem.

\left| z \right| = \sqrt{\Re(z)^2 + \Im(z)^2 }

where \Re \mbox{ and } \Im are the Real part and Imaginary part of z.

What your Wikipedia article is saying is that in these cases, which are typical, a magnitude is a non−negative real number.

> We can define the reals in terms of magnitude and we can define magnitude in terms of the reals.

Indeed we can, and both constructions are so closely related it makes no sense to claim that magnitude is easier. They are pretty much the same thing.

If we were in ancient Alexandria, it would have made a lot of sense to define the real numbers in terms of magnitude, because people had already defined magnitude. These days, we are not in that position, and people normally prefer to proced more directly to the real numbers, which are a field.

However, people do sometimes do things your way−−notably, Edmund Landau's famous "Foundations of Analysis" starts from positive integers, defines positive rationals, and then positive reals, or magnitudes, *before* introducing zero or negative numbers. This has the advantage that you can't divide by zero in your definitions because you haven't defined zero yet.

Which construction is more appropriate

> is a matter of putting the simpler concept beneath the more complicated concept. Magnitude is the less complicated of the two.

To you. I think they are more or less the same, and reals in some respects are less complicated, magnitude in other respects.

> Therefor defining magnitude from the reals is less meaningful than defining the reals from magnitude.

OK. But then you run into a problem: you haven't defined magnitude either. Since you haven't done that, you can't very well claim to have constructed the reals.

> Down at the bottom of these definitions are axioms. Can magnitude be axiomatic?

Absolutely.

The continuum concept is in there. Magnitude need not go

> into all of the number theory. It is primitive.

I'd be careful making that claim. Do you regard it as dispensible to
the concept of magnitude that any positive integer defines a magnitude?
What about the claim that for any given magnitude, there is always some
integer whose magnitude is greater?

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