Re: Attempts to Refute Cantor's Uncountability Proof?

Source: http://sci.tech−archive.net/Archive/sci.math/2006−07/msg01470.html

---

- *From*: "Dave L. Renfro" <renfr1dl@xxxxxxxxx>
- *Date*: 8 Jul 2006 20:29:18 −0700

Dave L. Renfro wrote:

> > > > At what point in your list will 1/3 be
> > > > reached? A rough
> > > > estimate would be acceptable.

Hatto von Aquitanien wrote:

> > > When I reach countable infinity.

Dave L. Renfro wrote:

> > The context is a list that has a first element, a second
> > element, a third element, and so on, for each positive
> > integer. There is no "countable infinite" position on
> > such a list.

Hatto von Aquitanien wrote:

> Every number I generate increments a counter by 1.

Fine. This means the counter will show '1', '2', '3', etc.
But at no point will the counter show "countable infinite".

Dave L. Renfro wrote:

> > To show that the positive rationals have the same cardinality
> > as the positive integers, you need to assign (in a unique way)
> > a certain positive rational number to '1', a certain positive

> rational number to '2', a certain positive rational number
> to '3', and so on, in such a manner that every positive
> rational number is used up. There is no "countable infinity"
> among the numbers '1', '2', '3', etc. −− you have to stay
> with the numbers '1', '2', '3', etc.

Hatto von Aquitanien wrote:

> And never mind that you are using integers up a lot faster than
> you are counting them. That simple fact right there is enough
> to make the whole proposition hard to accept. That really is
> the crux of the argument that methods created for dealing with
> finite sets are being abused by applying them to infinite sets.
> If I ask at any given point during this enumeration process
> what's the difference between the number of integers already
> counted, and the number consumed, the latter explodes. One
> typically considers such numerical behavior to be divergence.
> I guess the argument Cantor will give is that at any given
> point in the process, there is a bijective map, and then apply
> induction.

What about the fact that I can list (in the manner we're talking about) all the positive integers using the positive even integers? In what follows, the counting is being done by the positive even integers (left of the arrows) and the numbers being counted are the positive integers to the right of the arrows.

2−−>1 4−−>2 6−−>3 8−−>4 10−−>5 12−−>6 14−−>7 . . .

In this situation, I'm using up the even integers that I'm "counting with" much faster than the even integers that are among the numbers I'm counting. The (arithmetic) difference between the number of even integers counted vs. the number of even integers used up increases without bound (or "explodes", as you say).

Instead of even integers, if I use powers of 2 to count with, I can make the gaps increase fast enough so that we actually get the *ratios* between the number of those integers that I'm using to count with (i.e. the powers of 2) vs. the number of powers of 2 that get used up to increase without bound:

2−−>1 4−−>2 8−−>3 16−−>4 32−−>5 64−−>6 128−−>7 . . .

These are 1−1 correspondences, by the way. In the first case, each positive even integer n is matched with exactly one positive integer, namely n/2, and each positive integer m is matched with exactly one positive even integer, namely 2m. In the second case, each positive integer power of 2 is matched with exactly one positive integer, namely the exponent of 2 associated with the power of 2, and each positive integer m is matched with exactly one positive integer power of 2, namely 2^m.

With the right approach, one can use the positive integers "to count" the positive rational numbers. Although Cantor is credited with this, I don't think it is considered all that significant, and in fact I believe that Dedekind (who Cantor was in correspondence with during 1873−74 when Cantor was taking his first steps into this area) independently came up with such an approach. Cantor's importance comes from the fact that: (1) he first considered in a systematic way what the consequences might be for comparing infinite sets in this way (by 1−1 correspondences); (2) he managed to come up with a proof that there exists an infinite set that can't be "counted" by using the positive integers (namely, the real numbers); (3) he created an elaborate set of mathematical tools that proved very useful for solving problems, and establishing connections, in a wide variety of mathematical fields.

Dave L. Renfro

.