Re: An uncountable countable set

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Dik T. Winter schrieb:

> > And that is all you have to object against my proof? We have a fixed
> > scheme of transpositions.
>
> It is my objection to what you see as the similarity.

But it is not justified. If a fixed rule is given which determines for each rational when it will be included in the set ordered by magnitude (and if this is determined for every rational), then the order by magnitude is established. Remember: You believe in the existence of a well order, although the smallest rational larger than 1/2 and the largest rational smaller than 1/2 and so on do not appear.

> > But we could apply the transpositions even without any law, applying
> > only the rule that a pair which is already ordered shall not be treated
> > for a second time.
>
> I thought that was part of the rule (they are really conditional
> transpositions), so what?

"If, then"−decisions occur, just like in Cantor's diagonal proof. Yes, it was part of the rule. But the rule also includes a schedule which transpositions will be considered first. This schedule could be abolished. The transpositions could be chosen by chance as long as the order by magnitude was not yet achieved. If the infinite existed, then Tristram Shandy would complete his diary and he would complete the well−order by size too. From that you can obtain that the infinite does not and cannot exist.

> > Then we can apply countably many arbitrary
> > transpositions until all elements are ordered by magnitude. The limit
> > to which every path will lead is always the same: The set of rationals

> ordered by magnitude and by natural indices.

Again, you use the word "limit" here, without definition. How do you *define* the limit of a sequence of transpositions? How do you *define* the limit of the set on which the transpositions are applied?

The limit is the stable state where no further transpositions are applied, because there are no two elements remaining, which were not ordered by magnitude.

> But there is no limit process. The *only* criterion about manipulations
> on countable infinite sets is whether one can determine *precisely* at
> which natural number something happens: After how many steps in a
> well−order of |Q the fraction 4711/235537 will appear,

There are no "steps" in a well−order of Q. There is no notion of at which natural number something happens. A well−ordering of Q means a precise set of rules that determine the natural number to which a rational corresponds. It is your re−ordering that requires some notion of limit.

"Step" was meant here as counting from n to n+1. A precise set of rules is not yet established for the algebraic numbers, as far as I know.

> for instance, or
> in which line of Cantor's list a certain diagonal element will be
> placed and so on.

This makes no sense to me. In the first place, it is not Cantor's list, it is a given list.

The first list was given by him.

In the second place I have no idea what you mean with "diagonal element".

A digit of the diagonal number.

But what is known is that the n−th digital place of the diagonal is derived from the n−th element in the given list.

The question is in which line the 20th 5 will appear as a digit of the diagonal number, for instance.

> And I can determine *precisely* after how many steps
> the number 4711/235537 will be inserted in the order by magnitude with
> all of its predecessors of the initial well−order.

Indeed, you can. But now you are talking about steps. In the previous no steps were involved. So, you are talking about a sequential process, which was not he case in the previous things.

The well−ordering of the algebraic numbers without repetitions is also a sequential process. What is the problem of such a process if the sequences are determined? Even Cantor's diagonal proof is a sequential process because to find line number n you have to count from 1 to n − and counting is a sequential process. You cannot know line number n without knowing line number n−1.

A mapping that well−orders
the rationals is *not* a sequential process. The determination of the n−th digit of the diagonal from a given list is *not* a sequential process.

Wrong. You cannot know line number n without knowing line number n−1. But even if you were right, your argument would be void. If infinity would exist, then an infinite set could be exhausted by a definition like that of Cantor or that of mine.

> This is fixed and
> can be calculated for any rational number. Therefore all rational
> numbers are covered and will successively appear in the well−order. The
> argument that there remain always infinitely many other rationals is
> wrong, because by definiton the fate of each and every rational is
> determined and can be calculated.

Yes, for each rational number in the well−ordered list you can calculate the step when it comes in place in a numerically ordered segment of the rationals. But this does *not* mean that the final result is a well−order. Because at no time can you calculate the place where that rational number will be at the end. You need to show (at least) that there is a first element in the final ordering.

I show that infinity does not exist, because there will never be the first element of the order.

> But that is not interesting. It is easy to see that the diagonal is a
> sequence of the same sort as are the list entries. Whether they are
> real numbers is uninteresting.

But it is just that part that is interesting. Try the same with a sequence of algebraic numbers. You need to prove that what you get is also an algebraic number (and you cannot). So for algebraic numbers the proof fails. For real numbers the proof goes through, because you can prove that the resulting number is also a real number.

I proved in my special list even that the diagonal number is a rational. No, it was definitely not Cantor's intention to show that the diagonal is a real. He even emphasized the indepence of his arguing from real numbers: "Es läßt sich aber von jenem Satze ein viel einfacherer Beweis liefern, der unabhängig von der Betrachtung der Irrationalzahlen ist."

> Interesting is that such sequences are
> uncountable.

A sequence is *never* uncountable. By definition of the words sequence and uncountable in mathematics.

I said "sequences". The set of sequences is uncountable. It is the same when one somewhat sloppy says the real numbers are uncountable. You should reply then that a real number is never uncountable.

> But in order to prove that the diagonal differs from every entry, there
> a limit is required but not available.

No, it is just that place where a limit is not required. By definition, for every n in N, the n−th digit in the diagonal will be different from the n−th digit of the n−th element of the list. I do not see why limits are needed here.

The reason is obvious by the sequences 0.999... and 1.000... .

> The missing limit has not been remedied but has only been put aside. My
> objection remains: But that there is no satisfactory limit
> consideration becomes clear from the following: We know that 0.999... =
> 1.000...

Yes, that is because of the way those notations are *defined*. Those notations do not come out of thin air. They need definition before they can be used. And their definitions include limits.

Why do they if you do not see why limits are needed in sequences? These
are sequences and nothing else. Would they be different without your
"special definition"?

> This leads to the result that a change of 1 in the limit where
> the digit number goes to oo does not have the effect which would be
> required in order to distinguish the diagonal number from the list
> numbers.

I can make no sense of this.

Just because 0.999... and 1.000... are not equal by any "special
definition".

>
> > Arguing against Cantor's original papers is futile. But let me take one
> > point. You write:
> > > with only two different symbols w and m (which might be interpreted as
> > > binary representations but were not).
> > And you continue with taking them to be binary represention s. This is
> > dishonest. You state, explicitly, that they were not binary
> > representations, and argue against them as if they were binary
> > representations.
>
> Sind nämlich m und w irgend zwei einander ausschließende Charaktere,
> so betrachten wir den Inbegriff M von Elementen E = (x1, x2, ..., x?, ...),
> welche von unendlich vielen Koordinaten x1, x2, ... x?, ...
> abhängen, wo jede dieser Koordinaten entweder m oder w ist.
>
> I think one can safely interpret this sentence as describing binary
> numbers, though they were not called so.

I think we can safely interprete this sentence as *not* describing binary
numbers.

Cantor seems not to share your opinion. He extended his proof and
choose 0 and 1 instead of m and w. (Collected works, p. 279) I think
one can safely interpret this as digits in the binary system.

And Zermelo remarks that in order to apply Cantor's proof to the
continuum," braucht man noch den Nachweis, daß sich das Kontinuum
eineindeutig auf die Menge der formal verschiedenen Dualbrüche
abbilden läßt, obwohl doch jede dyadische Rationalzahl p/2^n nicht
eine, sondern zwei dyadische Darstellungen (mit lauter Nullen oder
lauter Einsen am Ende) gestattet." (p. 281)

> In the system Cantor describes elements terminating with
> (..., m, m, m, m...) are different from all elements terminating with
> (..., w, w, w, w...); this is not true in the binary numbers. So when
> you want to argue against this argument you should remain in the context
> he provides. The set Cantor describes (according to your quote) is the
> countably infinite product of a set of two elements. Quite different
> from the numbers in binary notation.

I did not say that Cantor's strings were binary numbers. I said that they might be *interpreted* as binary (dyadische, dual) representations, safely. Cantor and Zermelo at least did so.

Regards, WM

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