Re: An uncountable countable set

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In article <1152605227.458575.197240@xxxxxxxxxxxxxxxxxxxxxxxxxxx>
mueckenh@xxxxxxxxxxxxxxxxx writes:

> Dik T. Winter schrieb:

....

>> > Then we can apply countably many arbitrary
>> > transpositions until all elements are ordered by magnitude. The limit
>> > to which every path will lead is always the same: The set of rationals
>> > ordered by magnitude and by natural indices.
>>
>> Again, you use the word "limit" here, without definition. How do you
>> *define* the limit of a sequence of transpositions? How do you *define*
>> the limit of the set on which the transpositions are applied?

> The limit is the stable state where no further transpositions are
> applied, because there are no two elements remaining, which were not
> ordered by magnitude.

That makes no sense as a definition, as you will never reach that stable
state.

>> There are no "steps" in a well−order of Q. There is no notion of at
>> which natural number something happens. A well−ordering of Q means a
>> precise set of rules that determine the natural number to which a
>> rational corresponds. It is your re−ordering that requires some notion
>> of limit.

> "Step" was meant here as counting from n to n+1. A precise set of rules
> is not yet established for the algebraic numbers, as far as I know.

But it is for the rationals. And I think it can also be found for the
algebraic numbers, but it will take a bit more care.

> for instance, or
> in which line of Cantor's list a certain diagonal element will be
> placed and so on.

This makes no sense to me. In the first place, it is not Cantor's list, it is a given list.

The first list was given by him.

Ok, perhaps.

In the second place I have no idea what you mean with "diagonal element".

A digit of the diagonal number.

What do you mean with a sentence like "in which line of Cantor's list a certain digit of the diagonal number will be placed"?

But what is known is that the n−th digital place of the diagonal is derived from the n−th element in the given list.

The question is in which line the 20th 5 will appear as a digit of the diagonal number, for instance.

Well, an interesting question, perhaps, but irrelevant to the discussion.

> And I can determine *precisely* after how many steps
> the number 4711/235537 will be inserted in the order by magnitude with
> all of its predecessors of the initial well−order.

Indeed, you can. But now you are talking about steps. In the previous no steps were involved. So, you are talking about a sequential process, which was not he case in the previous things.

The well−ordering of the algebraic numbers without repetitions is also a sequential process. What is the problem of such a process if the sequences are determined? Even Cantor's diagonal proof is a sequential process because to find line number n you have to count from 1 to n −

> > and counting is a sequential process. You cannot know line number n
> > without knowing line number n−1.

The mapping from the naturals to the list gives the n−th element. It is
in the *definition* of the list.

> > > A mapping that well−orders
> > > the rationals is *not* a sequential process. The determination of the
> > > n−th digit of the diagonal from a given list is *not* a sequential
> > > process

> > Wrong. You cannot know line number n without knowing line number n−1.

The mapping gives that.

> > But even if you were right, your argument would be void. If infinity
> > would exist, then an infinite set could be exhausted by a definition
> > like that of Cantor or that of mine.

Makes no sense. The natural numbers can not be exhausted by a sequential
process.

> > > Yes, for each rational number in the well−ordered list you can calculate
> > > the step when it comes in place in a numerically ordered segment of the
> > > rationals. But this does *not* mean that the final result is a well−order
> > > Because at no time can you calculate the place where that rational number
> > > will be at the end. You need to show (at least) that there is a first
> > > element in the final ordering.

> > I show that infinity does not exist, because there will never be the
> > first element of the order.

No, you do not show anything of the sort. You only show that in the limit
well−order is destroyed (by some definition of limit). But there is no
problem with that. I have already shown (with definitions of limit) that
you can destroy well−order of the naturals by an infinite sequence of
transpositions. Nobody has a problem with that, as it is well known that
what is the case in the limit is not necessarily what is the case outside
the limit.

> But that is not interesting. It is easy to see that the diagonal is a
> sequence of the same sort as are the list entries. Whether they are
> real numbers is uninteresting.

But it is just that part that is interesting. Try the same with a sequence of algebraic numbers. You need to prove that what you get is also an algebraic number (and you cannot). So for algebraic numbers the proof fails. For real numbers the proof goes through, because you can prove that the resulting number is also a real number.

I proved in my special list even that the diagonal number is a rational.

I wonder whether it was a proof or just some handwaving.

> Interesting is that such sequences are
> uncountable.

A sequence is *never* uncountable. By definition of the words sequence and uncountable in mathematics.

I said "sequences".

Ah, I misread. Interesting that is just what the first version of Cantor's proof did show, without any reference to numbers.

> But in order to prove that the diagonal differs from every entry, there
> a limit is required but not available.

No, it is just that place where a limit is not required. By definition, for every n in N, the n−th digit in the diagonal will be different from the n−th digit of the n−th element of the list. I do not see why limits are needed here.

The reason is obvious by the sequences 0.999... and 1.000... .

See above. By definition of the notation (and without such a definition, such notations are meaningless in mathematics), both are equal to 1.

Yes, that is because of the way those notations are *defined*. Those notations do not come out of thin air. They need definition before they can be used. And their definitions include limits.

Why do they if you do not see why limits are needed in sequences? These
are sequences and nothing else. Would they be different without your
"special definition"?

I am lost. Mathematically a sequence of symbols like 0.999... makes no sense (as a number) unless there is a definition. The sequence makes perfect sense as a sequence. But as a sequence I can only state that 1.000... is not equal to 0.999.... Only when we want to interprete them as numbers we need a definition, and with the common definition those two are the same *as numbers*.

> This leads to the result that a change of 1 in the limit where
> the digit number goes to oo does not have the effect which would be
> required in order to distinguish the diagonal number from the list
> numbers.

I can make no sense of this.

Just because 0.999... and 1.000... are not equal by any "special definition".

Well, as strings of symbols they are not equal. Do you mean that?

I think we can safely interprete this sentence as *not* describing binary numbers.

Cantor seems not to share your opinion. He extended his proof and choose 0 and 1 instead of m and w. (Collected works, p. 279) I think one can safely interpret this as digits in the binary system.

I think you misread. "Man verstehe unter M den Inbegriff aller eindeutigen Funktionen f(x), welche nur die beide Werte 0 oder 1 annehmen, waehrend x alle reellen Werte, die >= 0 und <= 1 sind, durchlaeuft." No sense of digits in a binary system. This is used to prove that the set of such functions on [0,1] has a larger cardinality than the set of numbers in that range.

And Zermelo remarks that in order to apply Cantor's proof to the continuum," braucht man noch den Nachweis, da=DF sich das Kontinuum eineindeutig auf die Menge der formal verschiedenen Dualbr=FCche abbilden l=E4=DFt, obwohl doch jede dyadische Rationalzahl p/2^n nicht eine, sondern zwei dyadische Darstellungen (mit lauter Nullen oder lauter Einsen am Ende) gestattet." (p. 281)

Yes, and it is only at *this* point that we can look at them as binary numbers, but *at the same time* dual representations are taken in account.

> In the system Cantor describes elements terminating with (..., m, m, m, m...) are different from all elements terminating with (..., w, w, w, w...); this is not true in the binary numbers. So when you want to argue against this argument you should remain in the context he provides. The set Cantor describes (according to your quote) is the countably infinite product of a set of two elements. Quite different from the numbers in binary notation.

> I did not say that Cantor's strings were binary numbers. I said that they might be *interpreted* as binary (dyadische, dual) representations, safely. Cantor and Zermelo at least did so.

You can do so *if* you take dual representations in account. But at least with Cantor I do not find any evidence that he *did* interprete them as binary numbers.

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