

Re: Whats with this sequence?

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[Danny]

This sequence is the difference of each triangle number and the corresponding Fibonacci number.

e.g.

Starting with the first triangle number and the first Fibonacci f(1) number --

t--f = difference

$$1 - 1 = 0$$

$$3 - 1 = 2$$

$$6 - 2 = 4$$

$$10 - 3 = 7$$

$$15 - 5 = 10$$

etc.

Gives this resulting sequence --

0,2,4,7,10,13,15,15,11,0,-23,-66,-142,-272,
-490,-851,-1444,-2413,...

Note: the 10th term (0) which is $55 - 55 = t(10) - f(10) = 0$.

All fine and good but, why on the third delta row beneath this sequence the Fibonacci sequence appears?

Because it has to? ;-) Here's a conventional table of differences:

T 1 3 6 10 15 21 28 36 45 55 66 78 91

F 1 1 2 3 5 8 13 21 34 55 89 144 233

T-F 0 2 4 7 10 13 15 15 11 0 -23 -66 -142

d1 2 2 3 3 3 2 0 -4 -11 -23 -43 -76

d2 0 1 0 0 -1 -2 -4 -7 -12 -20 -33

d3 1 -1 0 -1 -1 -2 -3 -5 -8 -13

d4 -2 1 -1 0 -1 -1 -2 -3 -5

d5 3 -2 1 -1 0 -1 -1 -2

d6 -5 3 -2 1 -1 0 -1

d7 8 -5 3 -2 1 -1

d8 -13 8 -5 3 -2

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d9 21 -13 8 -5
d10 -34 21 -13
d11 55 -34

You can find the Fibonacci sequence all over the place in that, like the negation of it in the tail end of row d3, and with alternating signs in the first column starting at row d3; etc.

Except for the leading 1,1 the 3rd delta sequence is the Fibonacci sequence excluding the double entry of [1,1] at the beginning the following starts after the [1,1]--
0,1,1,2,3,5,8,13,21,34,55,89,144...

It just seems strange that this sequence would produce the Fibonacci sequence in one of its deltas!
Also why did it not list the triangle numbers instead on say the same or some other delta row?

I expect the mystery will go away if you compute similar difference tables for the triangle numbers /alone/, and again for the negation of the Fibonacci sequence alone. The difference table for T-F is just the sum of the difference table for T and the difference table for -F (that has nothing to do with T and F specifically: show that the difference table for A+B is the sum of the difference tables for A and for B, for any sequences A and B).

Note that the difference table for the triangle numbers is entirely zero from the third row onward:

```
2 3 4 5 6 7 8 9 10 11 12 13 14
1 1 1 1 1 1 1 1 1 1 1 1 1
0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0
0 0 0 0 0 0
0 0 0 0 0
0 0 0 0
0 0 0 0
0 0 0
0 0
0
```

OTOH, the difference table for the negation of the Fibonacci numbers "reproduces itself" in weird and wonderful ways because of the relation between the way Fibonacci numbers are computed and the way difference tables are constructed (one adds consecutive pairs, the other subtracts them):

0 -1 -1 -2 -3 -5 -8 -13 -21 -34 -55 -89 -144

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-1 0 -1 -1 -2 -3 -5 -8 -13 -21 -34 -55
1 -1 0 -1 -1 -2 -3 -5 -8 -13 -21
-2 1 -1 0 -1 -1 -2 -3 -5 -8
3 -2 1 -1 0 -1 -1 -2 -3
-5 3 -2 1 -1 0 -1 -1
8 -5 3 -2 1 -1 0
-13 8 -5 3 -2 1
21 -13 8 -5 3
-34 21 -13 8
55 -34 21
-89 55
144

Note that the T-F difference table is identical to the -F difference table (just above) from its 3rd row onward -- and it has to be because the T difference table is all zeroes from its 3rd row onward.

Similarly, if you take T-S for /any/ sequence S, the difference table for T-S will be identical to the difference table for S alone from the 3rd row onward. The only special thing about the triangle numbers here is that its differences eventually all become 0, and that's true of any sequence S where S_i can be expressed as a polynomial in i (in general, if it's a polynomial of degree N , all entries are 0 from difference row $N+1$ onward; since T_i can be expressed by a quadratic in i , its differences became 0 starting at difference row $2+1 = 3$).