covering maps in fiber bundles

Source: http://sci.tech−archive.net/Archive/sci.math/2006−08/msg00385.html

- *From*: Zenobe Odilon <zenobe@xxxxxxxxxxxx>

- *Date*: Wed, 09 Aug 2006 10:30:42 +0100

(This question may be related to a previous one on the universal cover of Grassmann, but the relation is not clear to me.)

I have a fiber bundle pi:E−−>B where E and B are smooth connected manifolds.

Does there always exist a smooth connected submanifold M of E with the following property, denoting by pi_M the restriction of pi to M:
for every x in B there exists an open neighborhood U such that the inverse image of U under pi_M is a union of at least one and finitely many mutually disjoint open sets of M each of which is mapped diffeomorphically onto U by pi_M?

(The example I have in mind is the Grassmann manifold. View it as O(n+k)/(O(n) x O(k)) or as ST(n+k,k)/GL(k) where ST(n+k,k) is the set of all full−rank (n+k)xk matrices. At least for ST(n+k,k)/GL(k) I know that there is no "M" as specified above with the additional requirement that M is invariant by the left action of O(n+k) on ST(n+k,k). But if I don't require this invariance, I don't see why such an M would not exist. I just don't know how to construct one.)

Thanks!
.