# Re: Is magnitude more fundamental than the real numbers?

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- *Date*: Thu, 10 Aug 2006 00:21:35 GMT

In article <1155123799.250626.317520@xxxxxxxxxxxxxxxxxxxxxxxxxxx> "Timothy Golden BandTechnology.com" <tttpppggg@xxxxxxxxx> writes:
....

> I believe that I have resolved our apparent disagreement. I've scrambled together a low grade reciprocal finder (results with large error) and sure enough the divisor counterexamples that you provide have no decent reciprocal. I've read the definition of zero divisor and it is fine though it does not discuss anything about the underlying structure of the system it is applied to. I suppose that is up to us and I am now curious what we'll see above p4 in this context.

Zero−divisors are indeed pat of the structure, but much can be said about zero−divisors without even investigating the underlying structure (when that is a ring, of course).

> The 'other axes' are actually a plane. It is the plane that is perpendicular to the identity axis that passes through the origin.

Yup, the set of obvious zero−divisors are, using coordinates (x1, x2, x3, x4), the plane x1 + x3 = x2 + x4 and the line x1 = x3 & x2 = x4.

> We've discussed it a bit before but now I can see it in the context of division. Just as a multiplication by a point on the identity axis removes information so does a multiplication by any point in this plane. Instead of squashing the information into a line it yields two dimensions of result, but still provides an irreversible effect on any object. In effect the object has been squashed into a plane and can never be 'unsquashed' informationally.

Indeed, the product of a zero−divisor and an arbitrary element is a zero−divisor. That is easy to prove when the product is commutative.

> We cannot
> get three dimensions back from two. There must be a theorem on
> dimensional reduction and zero divisors. It's clear in the
> informational context.

There is not necessarily dimensional reduction. There are rings where every element is a zero−divisor. But in general the zero−divisors form subspaces of lower dimensionality (assuming we have some dimensional algebra over the reals, which you have). But it is all quite tricky, because if a and b are zero−divisors, a + b is not necessarily a zero−divisor.

> The zero
> product is composed of one value from each of these parts, those parts
> being the identity axis and the plane orthogonal to it on the origin.
> There should be a theorem there as well. Upon excluding this plane as
> well as the line that I call the identity axis we can have division.

Indeed, you can have division with the elements that are not zero−divisors, because they all have multiplicative inverses. It is just like in linear algebra, where a matrix has an inverse if and only if the determinant is not equal to 0. But if you look at it, you will see that the structure of the set of matrices with determinant 0 is quite tricky.

> I suppose you are nearing an R x C style equivalent definition of the
> P4 product.

Pretty close, but not there yet. The line of zero−divisors should correspondent with R, and the plane with C. What remains to be done is finding the exact mapping. (That that is true follows because in R x C we have also two sets of zero−divisors, one 1D and one 2D, corresponding to R and C respectively. I think an exact mapping can be found.)

> As I scramble to follow along now I am wondering what you
> will find next. And what of P6? Is the zero divisor exclusion the
> identity axis and the points orthogonal to it through the origin?

There are many more zero−divisors in P6. I will explain, and that shows also the reason that I originally thought that Pn might have no zero−divisors if n is prime. Consider in P6:

(1, 1, 1, 0, 0, 0) * (1, 0, 0, 1, 0, 0) = 0
and
(1, 1, 0, 0, 0, 0) * (1, 0, 1, 0, 1, 0) = 0
These are obvious zero−divisors (you get the other obvious zero−divisors by circulating the coordinates). That we have two pairs of clearly different kinds is because 6 = 2 * 3. I do not know whether there are non−obvious zero−divisors (I am pretty sure, there are none in P4).

> What
> is a plane in P4 would be a 4D space in P6. This would be a ( 1, n−1 ) type of relation. That is consistent with the zero products, whose possibility space rises dimensionally.

What you will find (I think) in P6 is that there are four spaces of zero−divisors.

> If the Hopf, Milnor and Kervaire dimensional theorems apply then we should be able to get a zero divisor structure in P7 as well. But they suggest that P5 will work perfectly.

In P5 and P7 there are no obvious zero−divisors. So looking for them can be problematical. But starting at Pn, we can get an (n−1)− dimensional algebra over R by subtracting the last coordinate from all coordinates. The last coordinate now is 0. It is now an algebra over the reals of dimension n−1. So by Hopf, if it is a division algebra, n−1 should be a power of 2. We excluded already all non−prime n, now we can also exclude all primes that are not of the form 2^k + 1. But there is a stronger theorem by Hopf: every commutative division algebra over k has dimension 1 or 2. (H. Hopf, Ein topologischer Beitrag zur reellen Algebra, Comment. Math. Helv. 13 (1940), 219−239.) So we do not even need the theorems by Milnor and Kervaire. What they did show was that *any* division algebra over R has at most dimension 8, and that those algebras are precisely R, C, Q and O. (But Q is non−commitative and O is non−associative. If we get further we get the sedonians, but now there are zero−divisors.)

What remains is looking at P5 and see whether zero−divisors can be found (there must be some, otherwise it would be a division algebra, and that can not be the case by the theorem of Hopf.)

and so by the theorem they can only be a division algebra (that is, division does exist) if n−1 = 1, 2, 4 or 8.

> I'm not very good at linear algebra but I'll try to follow along. So far I don't see how you got the matrix in b. Is this a conjugation style of division?

No, it is just plain linear algebra notation to denote a set of linear
equations, which division in your systems is: solving sets of linear
equations.
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