Polysign Quotients

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- *From*: "Timothy Golden BandTechnology.com" <tttpppggg@xxxxxxxxx>
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Dik T. Winter wrote:

> In article <1155123799.250626.317520@xxxxxxxxxxxxxxxxxxxxxxxxxxx> "Timothy Golden BandTechnology.com" <tttpppggg@xxxxxxxxx> writes:
> ...
> > I believe that I have resolved our apparent disagreement. I've
> > scrambled together a low grade reciprocal finder (results with large
> > error) and sure enough the divisor counterexamples that you provide
> > have no decent reciprocal. I've read the definition of zero divisor and
> > it is fine though it does not discuss anything about the underlying
> > structure of the system it is applied to. I suppose that is up to us
> > and I am now curious what we'll see above p4 in this context.
>
> Zero−divisors are indeed pat of the structure, but much can be said
> about zero−divisors without even investigating the underlying
> structure (when that is a ring, of course).
>
> > The 'other axes' are actually a plane. It is the plane that is
> > perpendicular to the identity axis that passes through the origin.
>
> Yup, the set of obvious zero−divisors are, using coordinates
> (x1, x2, x3, x4), the plane x1 + x3 = x2 + x4 and the line
> x1 = x3 & x2 = x4.
>
> > We've discussed it a bit before but now I can see it in the context of
> > division. Just as a multiplication by a point on the identity axis
> > removes information so does a multiplication by any point in this
> > plane. Instead of squashing the information into a line it yields two
> > dimensions of result, but still provides an irreversible effect on any
> > object. In effect the object has been squashed into a plane and can
> > never be 'unsquashed' informationally.
>
> Indeed, the product of a zero−divisor and an arbitrary element is a
> zero−divisor. That is easy to prove when the product is commutative.
>
> > We cannot
> > get three dimensions back from two. There must be a theorem on
> > dimensional reduction and zero divisors. It's clear in the

> informational context.

There is not necessarily dimensional reduction. There are rings where every element is a zero−divisor. But in general the zero−divisors form subspaces of lower dimensionality (assuming we have some dimensional algebra over the reals, which you have). But it is all quite tricky, because if a and b are zero−divisors, a + b is not necessarily a zero−divisor.

Also it can be said that if a and b are zero divisors then their product is not necesarily zero.
For instance −1*1 and #1+1 are both zero divisors. But their product is −2*2.
This is where the structure matters, for it can be said that iff a is on the 1D part and b is on the 2D part that their product will be zero. This is all in the P4 context. The 1D part forces a result on its axis and the 2D part forces a result on its plane, yielding the origin. The two are inherently tied together. I think that this dimensional level is more appropriate a focus than the phrase 'zero divisor' for this phrase connotes a general phenomenon but without the structure it is too abstract. Each zero divisor loses its meaning without being attributed to its place in the structure. For P4 this is an axis and its orthogonal plane at the origin.
In P6 the value (0,0,1,1,0,0) does not intuit to be normal to (1,0,1,0,1,0) but it is according to my computer a right angle. So thus far the zero divisor concept laid out here maintains an orthogonal relation in P6 as well. In P6 the space orthogonal to the identity axis is a 4D object. But this is just pinning down a known behavior. The P5 challenge sounds much more interesting since there is a sliver of hope of proving an exception to Hopf.

> The zero
> product is composed of one value from each of these parts, those parts
> being the identity axis and the plane orthogonal to it on the origin.
> There should be a theorem there as well. Upon excluding this plane as
> well as the line that I call the identity axis we can have division.

Indeed, you can have division with the elements that are not zero−divisors, because they all have multiplicative inverses. It is just like in linear algebra, where a matrix has an inverse if and only if the determinant is not equal to 0. But if you look at it, you will see that the structure of the set of matrices with determinant 0 is quite tricky.

Yes, and you can have division with the elements that are zero divisors.

From the example above

(−2*2) / (−1*1) = #1+1 .

> I suppose you are nearing an R x C style equivalent definition of the
> P4 product.

Pretty close, but not there yet. The line of zero−divisors should correspondent with R, and the plane with C. What remains to be done is finding the exact mapping. (That that is true follows because in R x C we have also two sets of zero−divisors, one 1D and one 2D, corresponding to R and C respectively. I think an exact mapping can be found.)

Yeah. It sure looks that way. I have gotten very close with the independent products. That this product generates a self−similar error that could be removed with more products suggests that there is an exact answer lurking. Would mean is that there is an alternate way to develop the polysign system that will look a lot more like the Clifford algebra? That would be too big of a leap I suppose. Understanding division on P5 should help.

> As I scramble to follow along now I am wondering what you
> will find next. And what of P6? Is the zero divisor exclusion the
> identity axis and the points orthogonal to it through the origin?

There are many more zero−divisors in P6. I will explain, and that shows also the reason that I originally thought that Pn might have no zero− divisors if n is prime. Consider in P6:
(1, 1, 1, 0, 0, 0) * (1, 0, 0, 1, 0, 0) = 0
and
(1, 1, 0, 0, 0, 0) * (1, 0, 1, 0, 1, 0) = 0

Above I've verified that this is an orthogonal set of vectors.

These are obvious zero−divisors (you get the other obvious zero−divisors by circulating the coordinates). That we have two pairs of clearly different kinds is because 6 = 2 * 3. I do not know whether there are non−obvious zero−divisors (I am pretty sure, there are none in P4).

> What
> is a plane in P4 would be a 4D space in P6. This would be a (1, n−1)
> type of relation. That is consistent with the zero products, whose
> possibility space rises dimensionally.

What you will find (I think) in P6 is that there are four spaces of zero−divisors.

> If the Hopf, Milnor and Kervaire dimensional theorems apply then we
> should be able to get a zero divisor structure in P7 as well. But they
> suggest that P5 will work perfectly.

In P5 and P7 there are no obvious zero−divisors. So looking for them can be problematical. But starting at Pn, we can get an (n−1)−dimensional algebra over R by subtracting the last coordinate from all coordinates. The last coordinate now is 0. It is now an algebra over the reals of dimension n−1. So by Hopf, if it is a division algebra, n−1 should be a power of 2. We excluded already all non−prime n, now we can also exclude all primes that are not of the form 2^k + 1. But there is a stronger theorem by Hopf: every commutative division algebra over k has dimension 1 or 2. (H. Hopf, Ein topologischer Beitrag zur reellen Algebra, Comment. Math. Helv. 13 (1940), 219−239.) So we do not even need the theorems by Milnor and Kervaire. What they did show was that *any* division algebra over R has at most dimension 8, and that those algebras are precisely R, C, Q and O. (But Q is non−commitative and O is non−associative. If we get further we get the sedonians, but now there are zero−divisors.)

What remains is looking at P5 and see whether zero−divisors can be found (there must be some, otherwise it would be a division algebra, and that can not be the case by the theorem of Hopf.)

Yes, I've already acknowledged you above here on this. I think this is worth focusing on.
I still have not found the Hopf proof. Since you have carefully noted its publication I'll see what can be gotten in english. In twenty pages is it possible that this assumption has been made without evaluation?:
" The assumption that the square of a unit vector
is positive unity leads to an algebra whose characteristic
quantities
are non−associative. " − Cargill Gilston Knott
This is a quote of a quote from
http://en.wikipedia.org/wiki/Cargill_Gilston_Knott
I have not read any of his work but it may be that the polysign construction livens such fundamental debates. Dimensionality is produced differently under the polysign construction.
Squaring unit vectors in P4 yields a cone. Squaring its components will not yield a sensible distance. That requires use of cross terms as well.

and so by the theorem they can only be a division algebra (that is, division does exist) if n−1 = 1, 2, 4 or 8.

> I'm not very good at linear algebra but I'll try to follow along. So
> far I don't see how you got the matrix in b. Is this a conjugation
> style of division?

No, it is just plain linear algebra notation to denote a set of linear equations, which division in your systems is: solving sets of linear equations.

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dik t. winter, cwi, kruislaan 413, 1098 sj amsterdam, nederland, +31205924131
home: bovenover 215, 1025 jn amsterdam, nederland; http://www.cwi.nl/~dik/

One would hope that there is a means of doing the division out shorthand and this would imply a conjugate that yields the reciprocal indirectly. That this would be tied to the structure of the zero divisors is likely since they are what will break the division process. I still don't see how to do this yet the product is simply defined. Does the existence of zero divisors necessarily conflict with the notion of a shorthand division process?

−Tim

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