Re: complex fourier transform

Source: http://sci.tech−archive.net/Archive/sci.math/2006−08/msg02469.html

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- *Date*: 21 Aug 2006 17:57:58 −0700

Juryu wrote:

> Here I am with another problem studying for my complex variables qualifying exam.
>
> Problem:
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> ***
>
> The complex fourier transform
>
> Ff(w) = int_[from −infinity to infinity] exp(−2*pi*i*t*w) f(t)dt
>
> is defined, at least for real w, for any rapidly vanishing complex valued function f on the real line. Determine the Fourier transform of f(t) = exp (−t^2/2), using contour bending and the well−known fact that
>
> int_[from −infinity to infinity] exp(−t^2/2) dt = sqrt(2*pi).
>
> Determine the set of w in the complex plane for which the integral is absolutely convergent.
>
> ***
>
> First of all, I don't know what they mean by "contour bending", but I'm able to solve the first part by completing the square on the expontential, the answer is exp(−2*pi^2*w^2)*sqrt(2*pi).

I haven't heard of "contour bending" either, I suspect they mean starting with a contour and then expanding it to infinity or some such. Your method of completing the square doesn't quite work with a complex w because after you change the variables you end up with an integral of exp(−u^2/2) where u is not in the real line.

> For the second part, if I want it to be absolutely convergent, call w = x+iy and do the same for int_[from −infinity to infinity]

> abolute_value_of(exp(−2*pi*i*t*w) * exp(−t^2/2)) dt, and we get
> answer exp(2*pi^2*y^2)*sqrt(2*pi) < infinity, so it is absolutely
> convergent for ALL values of w?? Doesn't seem like an answer... Am I
> doing something wrong?

There is no "i" in the exponent after you take the absolute value so the previous result doesn't apply AFAICT.

−−
Jan Bielawski

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