Re: a proof for consideration

Source: http://sci.tech−archive.net/Archive/sci.math/2006−09/msg00347.html

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- *Date*: 2 Sep 2006 06:51:19 −0700

Proginoskes wrote:

> matt.zellman@xxxxxxxxx wrote:
>
>> so a couple of weeks ago, I posted here asking for help with a paper. I decided to go ahead and put what I have out here so people can look it over. I'll reiterate some of the concepts I went over in the previous thread, and then see if my reasoning is sound.
>
> Recently, I got a few free moments, and I looked up Richard Steinberg's paper "The Three−Color Problem" (to update and fix errors on my page about Steinberg's Conjecture), and I found that some of Zellman's concepts showed up there as well.
>
>> 1. k−chromatic Edge Replacement Subgraphs (^kERSs)
>>
>> a ^kERS is a k−chromatic graph that contains at least one pair of nonadjacent vertices for which no valid k−colorings exist when they are colored the same color.
>
> A couple of Russian mathematicians, V.A. Aksionov and L.S. Mel'nikov, called ^kERS's "quasi−edges" in a 1978 paper ("Essay on the theme: the three−color problem", Combinatorics, Colloquia Mathematica Societatis Janos Bolyai 18, 23−34).

Would it be more appropriate for me to switch to this term, since it predates mine? I think it is much clearer to include the chromatic number in the designation, because such subgraphs only really work when colored with a set number of colors.

>> The ^kERS as a whole functions in exactly the
>> same way as a single edge, and a k−chromatic graph can be transformed

G=>G' by replacing an edge with a ^kERS, a process I have called "expansion by edge−replacement."

The reverse process, replacing a ^kERS with an edge, I have termed "reduction by edge−replacement."

2. Boundary Points

Every graph has at least one set of vertices (of a size greater than or equal to the chromatic number k of the graph) for which no valid k−colorings exist when the vertices in the set are colored with less than k colors. Any such set of vertices is a set of "boundary points."

3. Basic k−chromatic Graphs

A primary basic k−chromatic graph is constructed by taking a basic (k−1)−chromatic graph and adding one vertex, which is then connected to an entire set of boundary points with edges or ^kERSs. A secondary basic k−chromatic graph is an expansion of a primary basic k−chromatic graph by edge−replacement. The basic 1−chromatic graph is a single vertex.

Every graph with chromatic number k contains at least one basic k−chromatic graph as a subgraph, and no basic (k+1) chromatic graphs as subgraphs.

This sounds a lot like a problem that Bjarne Toft raised in 1985:

PROBLEM. Suppose G is a (k+1)−colorable graph which does not contain K_(k+1). Does it follow that there are two vertices x and y and two k−colorable subgraphs G1 and G2, each containing x and y, such that:
(1) in any k−coloring of G1, x and y receive different colors, and
(2) in any k−coloring of G2, x and y receive the same color.

The converse is true for any k. This problem is true for k=2 but has been proven to be false if k >= 6. It is open for k=3, AFAIK.

Basically, the question is, "is there any combination of a ^kERS and an 'anti−^kERS' that would force k+1 colors, and does not contain a basic k−chromatic graph?"?

right?

The scope of the three−color problem is bounded only by the size of the graph. That is, consider the graph below (Figure A):

O−−−O−−−O−−−O−−−O

/|\ / \ / \ / \ /|
O−O O O O O |
\|/ \ / \ / \ / \|
O−−−O−−−O−−−O−−−O

This graph is 4−chromatic, as are all the graphs with the same end regions and different lengths of the same pattern in the middle. I could change something anywhere on the graph to make the chromatic number 3. For example, I could delete the leftmost vertex, I could add a vertex in the middle of the edge at the right end, or I could change any of the middle vertices to other configurations.

To put it succinctly, in order to determine the colorability of the graph, I am required to make an exhaustive analysis. No local analysis would guarantee that I find all the structures that determine the colorability of the graph.

Suppose, however, that using edge−replacement, we could reduce the graph to some configuration that could be analyzed locally. If we replace all the ^3ERSs with edges until there are no ^3ERSs left, we will be left with a graph that can be analyzed locally, in deterministic polynomial time. We simply have to go through the graph, and for each vertex, we look for all the edges connected to that vertex, and take note of which vertex is on the other end of each edge. Then we look for all the edges between those vertices, and build a subgraph from them. We can try to color this subgraph with two colors. If we can, then the graph may still be 3−colorable. If we can't, then the graph is not 3−colorable.

It certainly seems like a reasonable course of action, but it turns out that even identifying ^3ERSs in a graph is necessarily exponential over the inputs. Go back to the example graph shown above. If we take it back to the basic 3−chromatic graph it is constructed from, we are left with the graph below (Figure B):

O−−−O−−−O−−−O−−−O
/ \ / \ / \ / \ /|
O O O O O |
\ / \ / \ / \ / \|
O−−−O−−−O−−−O−−−O

It is an example of a graph in which a single ^3ERS has been iterated 4 times from a simple triangle. The ^3ERS by itself looks like this (Figure C):

1@−−−O2
\ /|
3O |
/ \|

4@−−−O5

(where the @s signify the endpoints of the edge that is being replaced,
and the vertices are numbered from 1−5 to help us out later)

Aksionov and Mel'nikov call the smaller graph a "building block".
Richard Steinberg's paper goes into more detail.

−−− Christopher Heckman

Where would be the best place to get ahold of these papers? What
exactly am I looking for?

In each even iteration of this ^3ERS, an edge can be constructed between the initial vertex 1, and vertex 5 of the second/fourth/sixth... iteration, without changing the colorability of the graph. Alternatively, an edge can be constructed between the initial vertex 4 and vertex 2 of that iteration. For odd iterations, they switch: initial vertex 1 can be connected to vertex 2 of the third/fifth/seventh... iteration, or initial vertex 4 can be connected to vertex 5 of that iteration.

Suppose that for each iteration, we construct one of these two edges. Such a construction across iterations creates a ^3ERS that cannot be reduced to another ^3ERS, but only straight to an edge. Each iteration adds 3 vertices and 7 edges to the graph. The extra edge we add makes it 3 and 8. We can represent which edge we have constructed at each iteration by simply making an ordered list: 14411144444... This particular example is equivalent to the graph represented by 41144411111, but not to the graph represented by 11411144444 or any other graph in the set. There are therefore at least 2^(i−2) unique possible ^3ERSs for each iteration i (i>1). Relating this to the size of the input (suppose our input is just the list of edges), the number of necessary tests for unique ^3ERSs for a graph with E edges must necessarily exceed (since our starting set of ^3ERSs is severely limited, as are the rules for construction):

2^(E/8−2) for E>16

Since without edge−replacement, the scope of the problem is unbounded, and therefore requires an exhaustive search, and with edge replacement, it requires a number of tests that is at least exponential over the size of the input, we are forced to conclude that the 3−coloring problem cannot be solved deterministically in polynomial time.

And, as a direct result, P is not equal to NP.

Is there anything I've missed?

Is this part of the proof new? or is it also covered in Steinberg or Askionov and Mel'nikov?

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