Re: JSH: The "Published" paper he dosen't what you to know about.

Source: http://sci.tech−archive.net/Archive/sci.math/2006−09/msg03660.html

---

- *From*: "marcus_b" <marcus_bruckner@xxxxxxxxx>
- *Date*: 18 Sep 2006 18:37:13 −0700

Sue wrote:

> Paper published and withdrawn paper by crank/crackpot James Harris exposes
> the "Over Iinterpertations of Galois Theory" by mathematicians.
> http://www.ne−plus−ultra.net/pubs/beckwith_factorization.pdf
>
> ADVANCED POLYNOMIAL FACTORIZATION
> James Harris
> A. W. Beckwith
> Department of Physics and
> Texas Center for Superconductivity
> University of Houston,
> Houston, Texas 77204−5005, USA,
> Fermi National Laboratory
> Batavia, IL 60510−050
>
> Abstract.
> Algebraic method for determining distribution of factors within a polynomial
> factorization, which breaks through what was seen as a barrier from over
> interpretations of Galois Theory
>
> Correspondence:
> A. W. Beckwith: projectbeckwith2@xxxxxxxxx
> PACS numbers : 02.10.−v, 02.10.De, 02.10.Ox , 02.20.Bb
>
> A.M.S. (MOS) Subject Classification Codes. 11R04,11R09
> KeyWords and Phrases. Polynomial factorization, Galois theory, Factorization
> lemma, Ring of algebraic integers
>
> Advanced Polynomial Factorization Approached.
>
> Determining the distribution of factors within irrational algebraic integers
> has long been considered impossible as it is not possible to do using Galois
> Theory. However a simple technique through the introduction of more
> variables makes it possible. To highlight the standard belief consider the
> algebraic integer roots of x^2 + x − 5.
>
> While you know that the algebraic integer factors are themselves factors of

5, can either not have non unit factors of 5? How do you know?

In looking to consider distribution of algebraic integer factors within a factorization I'll be using a more complicated example than x^2 + x ? 5.

This paper will show, using basic algebraic methods, that given the factorization, in the ring of algebraic integers,

65x^3 − 12x+ 1 = (a1x + 1)(a2x + 1)(a3x+ 1)

one of the a's is coprime to 5.

First I'll need a simple lemma to generalize beyond factors of a polynomial that are themselves polynomials.

Factorization Lemma:.
Given a factor g of a polynomial P(x), further defined as a factor for all x, which means that the value of g for a value 'a' of x is a factor of P(a), within the ring of algebraic integers, there exists r and c such that

g = r + c

where r =0, or varies as x varies, and c is a factor of the constant term P(0) and is itself constant.

Let x =0, then g must be a factor of P(0), so at that point c = g.

If when x does not equal 0, g = c, r =0. If when x does not equal 0, g = c there must exist r which varies with x. That is, r =g− c.
–

As an example consider sqrt(x +1) which is a non polynomial factor of x +1, and while there are an infinity of irrational solutions consider the rational solution at x =35.

Then I have sqrt(35 +1) = 6 = 5 + 1; therefore when x =35, g =6, r =5, and c =1. But for different values of x, g and r will vary, while c will not.

Primary Argument.

Given

65*x^3 −12*x+ 1 = (a1*x + 1)*(a2*x + 1)*(a3*x+ 1)

in the ring of algebraic integers. Let

P(m) = f^2*((m^3*f^4 − 3*m^2*f^2 + 3*m)*x^3 ? 3(?1 + m*f^2)*x*u^2 + u^3f)

Here f is a non unit, non zero algebraic integer coprime to 3 and x, and u a non unit, non zero algebraic integer coprime to f. Note P(m) has a factor that is f2.

That expression comes from expanding $(v^3+1)*x^3-3v*x*y^2+y^3$, using the substitutions

$v = -1+m*f^2$, and $y = u*f$,

where additional variables provide an additional degree of freedom.

Now consider the factorization

$P(m) = (a1*x + u*f)(a2*x + u*f)(a3*x + u*f)$

where multiplying out shows that

$a1*a2*a3 = m^3*f^6 - 3*m^2*f^4 + 3m*f^2 = f^2(m^3*f^4 - 3*m^2*f^2 + 3*m)$

so

$a1*a2*a3 = mf^2*(m^2*f^4 - 3m*f^2 + 3)$.

Therefore, at least one of the a's cannot be coprime to m, and at least one of the a's must equal 0 when m=0.

(Note: The a's are roots of a monic polynomial with algebraic integer coefficients so they are algebraic integers.)

Notice that the constant term P(0) is given by $P(0) = f^2*(3*x*u^2 + u^3*f)$ and also that $P(0)/f^2 = 3*x*u^2 + u^3*f$, which is coprime to f.

Then I have the factor of P(m), g1, where $g1 = a1*x+u*f$, where here I also have that a1 is not coprime to m.

From my factorization lemma, I have that, when m =0, g1 = c = u*f, meaning f is a factor of the constant term.

Therefore, exactly two of the a's equal 0, when m =0, to get the factor f^2 in the constant term P(0), while one must not equal 0, or f^3 would be the factor.

Now as noted before in general P(m) has a factor that is f^2, and separating that factor off, gives a constant term coprime to f; therefore, given g1 = a1*x + u*f where with m = 0, g1 gives a factor of f it must have that same factor in general, proving that two of the a's have a factor that is f.

Therefore, one factor is coprime to f.

Now letting m =1, f = sqrt(5) , where I can let u=1 as its value doesn't change the a's,

I have

(m^3*f^6−3*m^2*f^4 + 3*m)*x^3−3*(?1+ m*f^2)*x*u^2 + u^3 = 65*x^3−2*x+ 1

which may be more easily seen from using

v = −1 + m*f^2 = 4, y=1 with (v^3 +1)*x^3 ? 3*v*x*y^2 + y^3.

Therefore, with the factorization

65*x^3 − 12*x+ 1 = (a1*x + 1)(a2*x + 1)(a3*x+ 1)

one of the a's is coprime to 5, which shows where some of the algebraic integer factors distribute despite the factors being irrational.

Another version, different in detail, not co−authored by Beckwith, appears in a webzine called 'Chiaroscuro':

http://www.etienne.nu/isis/Chiaroscuro8.pdf

The date is July 2005. The first sentence is: "Over a hundred years ago subtle error crept into number theory."

A variant of Harris's deeply flawed argument involving constant terms of functions is still present.

Is this the version that was sent to the Annals of Mathematics?

Marcus.

.