

Re: a proof for consideration

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- *From:* "mimouni" <mimouni.mohamed@xxxxxxxxxx>
 - *Date:* 7 Oct 2006 05:15:42 -0700
-

Matt Zellman a écrit :

I finally got my hands on a copy of Steinberg's paper, and the key line of reasoning I use to make the proof that P is not equal to NP is not addressed in the paper. The focus of that paper is on finding local structures that would determine 3-colorability, while I demonstrate that such an approach cannot be satisfactory. Furthermore, I show that reducing the problem to one that is local in scope necessarily requires exponential time as well.

As far as I can tell, based on the Steinberg paper (which, admittedly, is 13 years old), the approach I used is novel. I haven't come across anything more recent that would suggest otherwise, either.

Matt Zellman wrote:

Matt Zellman wrote:

Proginoskes wrote:

Matt Zellman wrote:

Proginoskes wrote:

matt.zellman@xxxxxxxxxx
wrote:

so
a
couple
of
weeks
ago,
I
posted
here

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asking
for
help
with
a
paper.
I
decided
to
go
ahead
and
put
what
I
have
out
here
so
people
can
look
it
over.
I'll
reiterate
some
of
the
concepts
I
went
over
in
the
previous
thread,
and
then
see
if
my
reasoning
is
sound.

Recently, I
got a few
free
moments,

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and I looked
up Richard
Steinberg's
paper "The
Three-Color
Problem"
(to update
and fix
errors on
my page
about
Steinberg's
Conjecture),
and I found
that some of
Zellman's
concepts
showed up
there as
well.

1.
k-chromatic
Edge
Replacement
Subgraphs
(\wedge kERSs)

a
 \wedge kERS
is
a
k-chromatic
graph
that
contains
at
least
one
pair
of
nonadjacent
vertices
for
which
no
valid
k-colorings
exist
when

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they
are
colored
the
same
color.

A couple of
Russian
mathematicians,
V.A.
Aksionov
and L.S.
Mel'nikov,
called
"quasi-edges"
in a 1978
paper
("Essay on
the theme:
the
three-color
problem",
Combinatorics,
Colloquia
Mathematica
Societatis
Janos
Bolyai 18,
23-34).

Would it be more
appropriate for me to switch
to this term, since it
predates mine? I think it is
much clearer to include the
chromatic
number in the designation,
because such subgraphs
only really work when
colored with a set number of
colors.

Sticking with "quasi-edges" would cause
less confusion. However, if
calling them "k-quasi-edges" would be a

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nice compromise.

Sounds good to me. "k-quasi-edges" it is.

The
 k -ERS
as
a
whole
functions
in
exactly
the
same
way
as
a
single
edge,
and
a
 k -chromatic
graph
can
be
transformed
 $G \Rightarrow G'$
by
replacing
an
edge
with
a
 k -ERS,
a
process
I
have
called
"expansion
by
edge-replacement."

The
reverse
process,
replacing

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a
kERS
with
an
edge,
I
have
termed
"reduction
by
edge-replacement."

2.
Boundary
Points

Every
graph
has
at
least
one
set
of
vertices
(of
a
size
greater
than
or
equal
to
the
chromatic
number
k
of
the
graph)
for
which
no
valid
k-colorings
exist
when
the
vertices
in
the

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set
are
colored
with
less
than
k
colors.

Any
such
set
of
vertices
is
a
set
of
"boundary
points."

3.
Basic
k-chromatic
Graphs

A
primary
basic
k-chromatic
graph
is
constructed
by
taking
a
basic
(k-1)-chromatic
graph
and
adding
one
vertex,
which
is
then
connected
to
an
entire
set
of

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boundary
points
with
edges
or
 k -ERSs.
A
secondary
basic
 k -chromatic
graph
is
an
expansion
of
a
primary
basic
 k -chromatic
graph
by
edge-replacement.
The
basic
1-chromatic
graph
is
a
single
vertex.

Every
graph
with
chromatic
number
 k
contains
at
least
one
basic
 k -chromatic
graph
as
a
subgraph,
and
no
basic
 $(k+1)$

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chromatic
graphs
as
subgraphs.

This sounds
a lot like a
problem
that Bjarne
Toft raised
in 1985:

PROBLEM.
Suppose G
is a
 $(k+1)$ -colorable
graph which
does not
contain
 K_{k+1} .
Does it
follow that
there are
two vertices
 x and y and
two
 k -colorable
subgraphs
 G_1 and G_2 ,
each
containing x
and y , such
that:
(1) in any
 k -coloring
of G_1 , x and
 y receive
different
colors, and
(2) in any
 k -coloring
of G_2 , x and
 y receive
the same
color.

The
converse is
true for any
 k . This

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problem is
true for $k=2$
but has
been proven
to be false if
 $k \geq 6$. It is
open for
 $k=3$,
AFAIK.

Basically, the question is,
"is there any combination of
a k -ERS and an
'anti- k -ERS' that would
force $k+1$ colors, and does
not contain a basic
 k -chromatic graph?"?

right?

Yes. Although here, all "boundary sets"
would have size two.

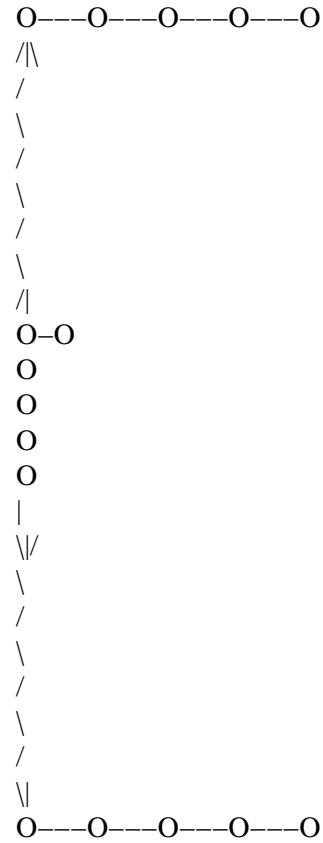
interesting. I think I can prove that the answer to this
question is
"no" for $k=3$.

should I go ahead and attempt this proof? or is it even necessary?

The
scope
of
the
three-color
problem
is
bounded
only
by
the
size
of

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the
graph.
That
is,
consider
the
graph
below
(Figure
A):



This
graph
is
4-chromatic,
as
are
all
the
graphs
with
the
same
end
regions
and

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different
lengths
of
the
same
pattern
in
the
middle.
I
could
change
something
anywhere
on
the
graph
to
make
the
chromatic
number
3.
For
example,
I
could
delete
the
leftmost
vertex,
I
could
add
a
vertex
in
the
middle
of
the
edge
at
the
right
end,
or
I
could
change
any

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of
the
middle
vertices
to
other
configurations.

To
put
it
succinctly,
in
order
to
determine
the
colorability
of
the
graph,

I
am
required
to
make
an
exhaustive
analysis.

No
local
analysis
would
guarantee
that

I
find
all
the
structures
that
determine
the
colorability
of
the
graph.

Suppose,

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however,
that
using
edge-replacement,
we
could
reduce
the
graph
to
some
configuration
that
could
be
analyzed
locally.
If
we
replace
all
the
 \wedge^3 ERSs
with
edges
until
there
are
no
 \wedge^3 ERSs
left,
we
will
be
left
with
a
graph
that
can
be
analyzed
locally,
in
deterministic
polynomial
time.
We
simply
have
to

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go
through
the
graph,
and
for
each
vertex,
we
look
for
all
the
edges
connected
to
that
vertex,
and
take
note
of
which
vertex
is
on
the
other
end
of
each
edge.
Then
we
look
for
all
the
edges
between
those
vertices,
and
build
a
subgraph
from
them.
We
can
try

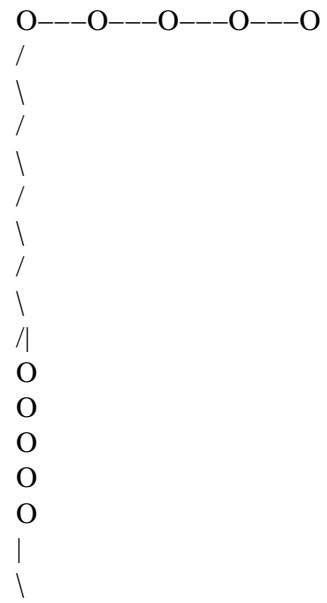
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to
color
this
subgraph
with
two
colors.
If
we
can,
then
the
graph
may
still
be
3-colorable.
If
we
can't,
then
the
graph
is
not
3-colorable.

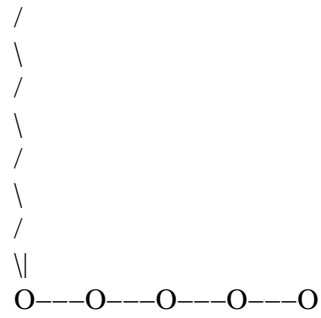
It
certainly
seems
like
a
reasonable
course
of
action,
but
it
turns
out
that
even
identifying
 \wedge^3 ERSs
in
a
graph
is
necessarily
exponential
over

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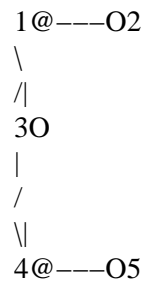
the
inputs.
Go
back
to
the
example
graph
shown
above.
If
we
take
it
back
to
the
basic
3-chromatic
graph
it
is
constructed
from,
we
are
left
with
the
graph
below
(Figure
B):



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It is an example of a graph in which a single ³ERS has been iterated 4 times from a simple triangle. The ³ERS by itself looks like this (Figure C):



(where

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the
@s
signify
the
endpoints
of
the
edge
that
is
being
replaced,
and
the
vertices
are
numbered
from
1-5
to
help
us
out
later)

Aksionov
and
Mel'nikov
call the
smaller
graph a
"building
block".
Richard
Steinberg's
paper goes
into more
detail.

Where would be the best
place to get ahold of these
papers? What
exactly am I looking for?

The Steinberg paper is in a book called
Quo Vadis, Graph Theory?,
which should be in your local university
library (call number QA166 .Q6

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1993). (This book is about 400 pages long, but Steinberg's article only consists of pages 211–248.) I would think it's too old to be available electronically. Steinberg might still have preprints or reprints, but I doubt it. (His webpage is <http://www.jbs.cam.ac.uk/people/faculty/steinberg.html>).

I will check this out the next chance I get.

I finally located a copy and am waiting for it to come in. There isn't a library in the area that has it.

In
each
even
iteration
of
this
 \wedge 3ERS,
an
edge
can
be
constructed
between
the
initial
vertex
1,
and
vertex
5
of
the
second/fourth/sixth...
iteration,
without
changing
the
colorability
of
the
graph.

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Alternatively,
an
edge
can
be
constructed
between
the
initial
vertex
4
and
vertex
2
of
that
iteration.
For
odd
iterations,
they
switch:
initial
vertex
1
can
be
connected
to
vertex
2
of
the
third/fifth/seventh...
iteration,
or
initial
vertex
4
can
be
connected
to
vertex
5
of
that
iteration.

Suppose
that

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for
each
iteration,
we
construct
one
of
these
two
edges.
Such
a
construction
across
iterations
creates
a
 Δ ERS
that
cannot
be
reduced
to
another
 Δ ERS,
but
only
straight
to
an
edge.
Each
iteration
adds
3
vertices
and
7
edges
to
the
graph.
The
extra
edge
we
add
makes
it
3
and

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8.
We
can
represent
which
edge
we
have
constructed
at
each
iteration
by
simply
making
an
ordered
list:
14411144444...
This
particular
example
is
equivalent
to
the
graph
represented
by
41144411111,
but
not
to
the
graph
represented
by
11411144444
or
any
other
graph
in
the
set.
There
are
therefore
at
least
 $2^{(i-2)}$

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unique
possible
 \wedge^3 ERSs
for
each
iteration
 i
($i > 1$).
Relating
this
to
the
size
of
the
input
(suppose
our
input
is
just
the
list
of
edges),
the
number
of
necessary
tests
for
unique
 \wedge^3 ERSs
for
a
graph
with
 E
edges
must
necessarily
exceed
(since
our
starting
set
of
 \wedge^3 ERSs
is
severely
limited,

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as
are
the
rules
for
construction):

$2^{(E/8-2)}$
for
 $E > 16$

Since
without
edge-replacement,
the
scope
of
the
problem
is
unbounded,
and
therefore
requires
an
exhaustive
search,
and
with
edge
replacement,
it
requires
a
number
of
tests
that
is
at
least
exponential
over
the
size
of
the
input,
we
are

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forced
to
conclude
that
the
3-coloring
problem
cannot
be
solved
deterministically
in
polynomial
time.

And,
as
a
direct
result,
P
is
not
equal
to
NP.

Is
there
anything
I've
missed?

Is this part of the proof
new? or is it also covered in
Steinberg or
Askionov and Mel'nikov?

This last part isn't, but Steinberg usually
only summarizes results.
The reference list has 128 papers on it.

—— Christopher Heckman

Oh, fun, a scavenger hunt! ;-)

~Matt Zellman

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As a footnote, I realize I never actually stated the purpose of the exhaustive search in the proof, which leaves a rather significant disconnect for people that don't make the connection automatically. What we are searching for are the new points that were added in the construction of a basic 4-chromatic graph. In a sense, we are seeing whether the vertices directly connected to some particular vertex are a set of boundary points of a basic 3-chromatic graph (or really, any 3-chromatic graph, which if the conjecture about basic 3-chromatic graphs being inherent in all graphs of chromatic number 3 is true, amounts to the same thing). The scope of the search is the number of vertices removed from the initial vertex that we have to examine.

If it is necessary, I can prove that this is the most efficient algorithm possible (other than guess-and-check, which may actually be faster—as is the case for 2-coloring), though I have a hunch that this statement may be equivalent to—or at least follow quickly from—what was proved by Razborov and Rudich regarding "natural proofs."

does Razborov and Rudich's proof actually imply this, or is it wishful thinking on my part?

Hello Sirs,

Here an algorithm for 3 colors:

Either G a planar graph which A 3-cliques as cliques maximum, one notes

each 3-cliques by triangle.

G contains several triangles, to place the one to dimension others, or to separate by vertex.

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1) Zone of obligatory colour application simple:

If one gives to a triangle colors 1,2 and 3, one can find vertex which will be coloured in an obligatory way, after having all the zones there is the following result:

A– There has is a zone with 4 colors, that applies that G cannot be colour with 3 colors.

B– There N are no zones with 4 colors, one cannot nothing say...

2) Zone of made up obligatory colour application:

A zone is known as made up if it contains 2 or more of the simple zones

in such manner that the colour application of zone I obliges the colour application of

zone J, after having all the zones one has the following result:

A– There has is a zone with 4 colors, that applies that G cannot be colour with 3 colors.

B– There N are no zones with 4 colors, then G is to colour with 3 colors.

the number of the triangles in this case, compared to the numbers of the vertex is polynomial: $t = (n-3) * 2$.

Two simple zones cannot thus have the same triangle the number of the zones is also polynomial.

Suppose that G is coloured by 4 colors, in this case a vertex (4) is assistant at the three vertex (1, 2 and 3). If 1 is in Z1, 2 in Z2, and 3 in Z3; since the vertex are coloured forcing then there is a relation enters to, the 3 zones form a zone made up.

Remain the case general, if the graph has 4-cliques then it could not be coloured by 3 colors.

.