

Re: a proof for consideration

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- *From:* "Proginoskes" <CHeckman@xxxxxxxx>
 - *Date:* 7 Oct 2006 23:50:43 -0700
-

mimouni wrote:

Matt Zellman a écrit :

I finally got my hands on a copy of Steinberg's paper, and the key line of reasoning I use to make the proof that P is not equal to NP is not addressed in the paper. The focus of that paper is on finding local structures that would determine 3-colorability, while I demonstrate that such an approach cannot be satisfactory. Furthermore, I show that reducing the problem to one that is local in scope necessarily requires exponential time as well.

As far as I can tell, based on the Steinberg paper (which, admittedly, is 13 years old), the approach I used is novel. I haven't come across anything more recent that would suggest otherwise, either.

Matt Zellman wrote:

Matt Zellman wrote:

Proginoskes wrote:

Matt Zellman wrote:

Proginoskes
wrote:

matt.zellman@xxxxxxxx
wrote:

so
a
couple
of
weeks
ago,

Re: a proof for consideration

I
posted
here
asking
for
help
with
a
paper.
I
decided
to
go
ahead
and
put
what
I
have
out
here
so
people
can
look
it
over.
I'll
reiterate
some
of
the
concepts
I
went
over
in
the
previous
thread,
and
then
see
if
my
reasoning
is
sound.

Recently,

Re: a proof for consideration

I
got
a
few
free
moments,
and
I
looked
up
Richard
Steinberg's
paper
"The
Three-Color
Problem"
(to
update
and
fix
errors
on
my
page
about
Steinberg's
Conjecture),
and
I
found
that
some
of
Zellman's
concepts
showed
up
there
as
well.

1.
k-chromatic
Edge
Replacement
Subgraphs
(k ERSs)

a
 k ERS

Re: a proof for consideration

is
a
k-chromatic
graph
that
contains
at
least
one
pair
of
nonadjacent
vertices
for
which
no
valid
k-colorings
exist
when
they
are
colored
the
same
color.

A
couple
of
Russian
mathematicians,
V.A.
Aksionov
and
L.S.
Mel'nikov,
called
^kERS's
"quasi-edges"
in
a
1978
paper
("Essay
on
the
theme:
the
three-color

Re: a proof for consideration

problem",
Combinatorics,
Colloquia
Mathematica
Societatis
Janos
Bolyai
18,
23–34).

Would it be
more
appropriate
for me to
switch to
this term,
since it
predates
mine? I
think it is
much
clearer to
include the
chromatic
number in
the
designation,
because
such
subgraphs
only really
work when
colored
with a set
number of
colors.

Sticking with "quasi-edges"
would cause less confusion.
However, if
calling them
"k-quasi-edges" would be a
nice compromise.

Sounds good to me. "k-quasi-edges" it is.

Re: a proof for consideration

The
 k -ERS
as
a
whole
functions
in
exactly
the
same
way
as
a
single
edge,
and
a
 k -chromatic
graph
can
be
transformed
 $G \Rightarrow G'$
by
replacing
an
edge
with
a
 k -ERS,
a
process
I
have
called
"expansion
by
edge-replacement."

The
reverse
process,
replacing
a
 k -ERS
with
an
edge,
I

Re: a proof for consideration

have
termed
"reduction
by
edge-replacement."

2.
Boundary
Points

Every
graph
has
at
least
one
set
of
vertices
(of
a
size
greater
than
or
equal
to
the
chromatic
number
 k
of
the
graph)
for
which
no
valid
 k -colorings
exist
when
the
vertices
in
the
set
are
colored
with
less
than

Re: a proof for consideration

k
colors.
Any
such
set
of
vertices
is
a
set
of
"boundary
points."

3.
Basic
k-chromatic
Graphs

A
primary
basic
k-chromatic
graph
is
constructed
by
taking
a
basic
(k-1)-chromatic
graph
and
adding
one
vertex,
which
is
then
connected
to
an
entire
set
of
boundary
points
with
edges
or
 k -ERSs.

Re: a proof for consideration

A
secondary
basic
k-chromatic
graph
is
an
expansion
of
a
primary
basic
k-chromatic
graph
by
edge-replacement.
The
basic
1-chromatic
graph
is
a
single
vertex.

Every
graph
with
chromatic
number
k
contains
at
least
one
basic
k-chromatic
graph
as
a
subgraph,
and
no
basic
(k+1)
chromatic
graphs
as
subgraphs.

Re: a proof for consideration

This
sounds
a
lot
like
a
problem
that
Bjarne
Toft
raised
in
1985:

PROBLEM.
Suppose
G
is
a
(k+1)-colorable
graph
which
does
not
contain
 K_{k+1} .
Does
it
follow
that
there
are
two
vertices
x
and
y
and
two
k-colorable
subgraphs
G1
and
G2,
each
containing
x
and
y,
such

Re: a proof for consideration

that:
(1)
in
any
k-coloring
of
G1,
x
and
y
receive
different
colors,
and
(2)
in
any
k-coloring
of
G2,
x
and
y
receive
the
same
color.

The
converse
is
true
for
any
k.
This
problem
is
true
for
k=2
but
has
been
proven
to
be
false
if
k
>=

Re: a proof for consideration

6.
It
is
open
for
 $k=3$,
AFAIK.

Basically,
the question
is, "is there
any
combination
of a k -ERS
and an
'anti- k -ERS'
that would
force $k+1$
colors, and
does not
contain a
basic
 k -chromatic
graph?"?

right?

Yes. Although here, all
"boundary sets" would have
size two.

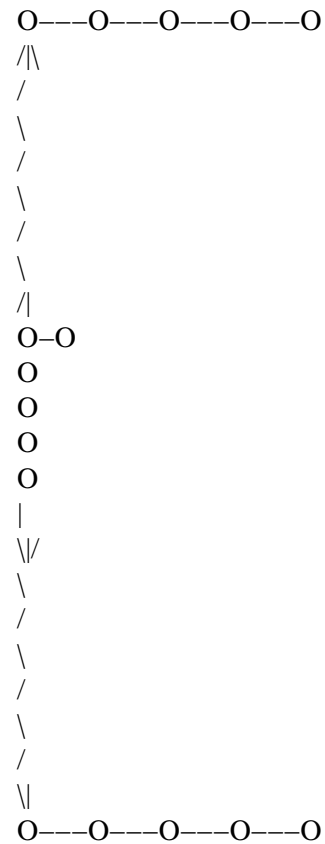
interesting. I think I can prove that the
answer to this question is
"no" for $k=3$.

should I go ahead and attempt this proof? or is it even
necessary?

The
scope
of
the

Re: a proof for consideration

three-color
problem
is
bounded
only
by
the
size
of
the
graph.
That
is,
consider
the
graph
below
(Figure
A):



This
graph
is
4-chromatic,
as
are

Re: a proof for consideration

all
the
graphs
with
the
same
end
regions
and
different
lengths
of
the
same
pattern
in
the
middle.
I
could
change
something
anywhere
on
the
graph
to
make
the
chromatic
number
3.
For
example,
I
could
delete
the
leftmost
vertex,
I
could
add
a
vertex
in
the
middle
of
the
edge

Re: a proof for consideration

at
the
right
end,
or
I
could
change
any
of
the
middle
vertices
to
other
configurations.

To
put
it
succinctly,
in
order
to
determine
the
colorability
of
the
graph,
I
am
required
to
make
an
exhaustive
analysis.
No
local
analysis
would
guarantee
that
I
find
all
the
structures
that
determine

Re: a proof for consideration

the
colorability
of
the
graph.

Suppose,
however,
that
using
edge-replacement,
we
could
reduce
the
graph
to
some
configuration
that
could
be
analyzed
locally.
If
we
replace
all
the
 $\wedge^3\text{ERSs}$
with
edges
until
there
are
no
 $\wedge^3\text{ERSs}$
left,
we
will
be
left
with
a
graph
that
can
be
analyzed

Re: a proof for consideration

locally,
in
deterministic
polynomial
time.
We
simply
have
to
go
through
the
graph,
and
for
each
vertex,
we
look
for
all
the
edges
connected
to
that
vertex,
and
take
note
of
which
vertex
is
on
the
other
end
of
each
edge.
Then
we
look
for
all
the
edges
between
those
vertices,

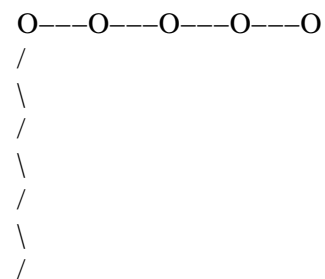
Re: a proof for consideration

and
build
a
subgraph
from
them.
We
can
try
to
color
this
subgraph
with
two
colors.
If
we
can,
then
the
graph
may
still
be
3-colorable.
If
we
can't,
then
the
graph
is
not
3-colorable.

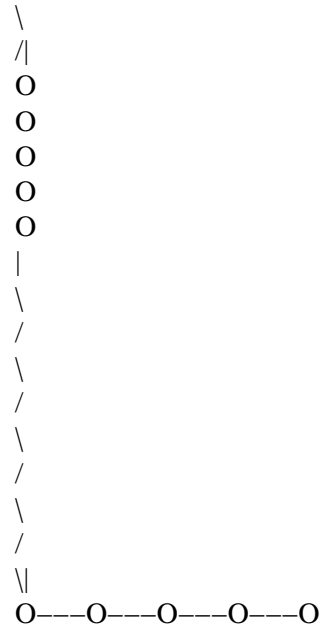
It
certainly
seems
like
a
reasonable
course
of
action,
but
it
turns
out
that
even

Re: a proof for consideration

identifying
^3ERSs
in
a
graph
is
necessarily
exponential
over
the
inputs.
Go
back
to
the
example
graph
shown
above.
If
we
take
it
back
to
the
basic
3-chromatic
graph
it
is
constructed
from,
we
are
left
with
the
graph
below
(Figure
B):



Re: a proof for consideration



It is an example of a graph in which a single Δ^3 ERS has been iterated 4 times from a simple triangle. The Δ^3 ERS by itself looks like this (Figure C):

1@---O2

Re: a proof for consideration

\
/
30
|
/
\
4@---05

(where
the
@s
signify
the
endpoints
of
the
edge
that
is
being
replaced,
and
the
vertices
are
numbered
from
1-5
to
help
us
out
later)

Aksionov
and
Mel'nikov
call
the
smaller
graph
a
"building
block".
Richard
Steinberg's
paper
goes
into
more

Re: a proof for consideration

detail.

Where
would be
the best
place to get
ahold of
these
papers?
What
exactly am I
looking for?

The Steinberg paper is in a book called *Quo Vadis, Graph Theory?*, which should be in your local university library (call number QA166 .Q6 1993). (This book is about 400 pages long, but Steinberg's article only consists of pages 211–248.) I would think it's too old to be available electronically. Steinberg *might* still have preprints or reprints, but I doubt it. (His webpage is <http://www.jbs.cam.ac.uk/people/faculty/steinbergr.html>).

I will check this out the next chance I get.

I finally located a copy and am waiting for it to come in. There isn't a library in the area that has it.

In
each
even
iteration
of
this

Re: a proof for consideration

$\wedge^3ERS,$
an
edge
can
be
constructed
between
the
initial
vertex
1,
and
vertex
5
of
the
second/fourth/sixth...
iteration,
without
changing
the
colorability
of
the
graph.
Alternatively,
an
edge
can
be
constructed
between
the
initial
vertex
4
and
vertex
2
of
that
iteration.
For
odd
iterations,
they
switch:
initial
vertex
1
can

Re: a proof for consideration

be
connected
to
vertex
2
of
the
third/fifth/seventh...
iteration,
or
initial
vertex
4
can
be
connected
to
vertex
5
of
that
iteration.

Suppose
that
for
each
iteration,
we
construct
one
of
these
two
edges.
Such
a
construction
across
iterations
creates
a
 \wedge^3ERS
that
cannot
be
reduced
to
another
 \wedge^3ERS ,
but

Re: a proof for consideration

only
straight
to
an
edge.
Each
iteration
adds
3
vertices
and
7
edges
to
the
graph.
The
extra
edge
we
add
makes
it
3
and
8.
We
can
represent
which
edge
we
have
constructed
at
each
iteration
by
simply
making
an
ordered
list:
14411144444...
This
particular
example
is
equivalent
to
the

Re: a proof for consideration

graph
represented
by
41144411111,
but
not
to
the
graph
represented
by
11411144444
or
any
other
graph
in
the
set.
There
are
therefore
at
least
 $2^{(i-2)}$
unique
possible
 $\wedge 3$ ERSs
for
each
iteration
 i
($i > 1$).
Relating
this
to
the
size
of
the
input
(suppose
our
input
is
just
the
list
of
edges),
the

Re: a proof for consideration

number
of
necessary
tests
for
unique
 \wedge^3ERSs
for
a
graph
with
E
edges
must
necessarily
exceed
(since
our
starting
set
of
 \wedge^3ERSs
is
severely
limited,
as
are
the
rules
for
construction):

$2^{(E/8-2)}$
for
 $E > 16$

Since
without
edge-replacement,
the
scope
of
the
problem
is
unbounded,
and
therefore
requires
an

Re: a proof for consideration

exhaustive
search,
and
with
edge
replacement,
it
requires
a
number
of
tests
that
is
at
least
exponential
over
the
size
of
the
input,
we
are
forced
to
conclude
that
the
3-coloring
problem
cannot
be
solved
deterministically
in
polynomial
time.

And,
as
a
direct
result,
P
is
not
equal
to
NP.

Re: a proof for consideration

Is
there
anything
I've
missed?

Is this part
of the proof
new? or is it
also
covered in
Steinberg or
Askionov
and
Mel'nikov?

This last part isn't, but
Steinberg usually only
summarizes results.
The reference list has 128
papers on it.

---- Christopher Heckman

Oh, fun, a scavenger hunt! ;-)

~Matt Zellman

As a footnote, I realize I never actually
stated the purpose of the
exhaustive search in the proof, which leaves
a rather significant
disconnect for people that don't make the
connection automatically.
What we are searching for are the new
points that were added in the
construction of a basic 4-chromatic graph.
In a sense, we are seeing
whether the vertices directly connected to
some particular vertex are a
set of boundary points of a basic
3-chromatic graph (or really, any
3-chromatic graph, which if the conjecture
about basic 3-chromatic

Re: a proof for consideration

graphs being inherent in all graphs of chromatic number 3 is true, amounts to the same thing). The scope of the search is the number of vertices removed from the initial vertex that we have to examine.

If it is necessary, I can prove that this is the most efficient algorithm possible (other than guess-and-check, which may actually be faster—as is the case for 2-coloring), though I have a hunch that this statement may be equivalent to—or at least follow quickly from—what was proved by Razborov and Rudich regarding "natural proofs."

does Razborov and Rudich's proof actually imply this, or is it wishful thinking on my part?

Hello Sirs,

Here an algorithm for 3 colors:

Either G a planar graph which A 3-cliques as cliques maximum, one notes each 3-cliques by triangle.

G contains several triangles, to place the one to dimension others, or to separate by vertex.

1) Zone of obligatory colour application simple:

If one gives to a triangle colors 1,2 and 3, one can find vertex which will be coloured in an obligatory way, after having all the zones there is the following result:

A– There has is a zone with 4 colors, that applies that G cannot be colour with 3 colors.

B– There N are no zones with 4 colors, one cannot nothing say...

2) Zone of made up obligatory colour application:

A zone is known as made up if it contains 2 or more of the simple zones

in such manner that the colour application of zone I obliges the colour application of

zone J, after having all the zones one has the following result:

A– There has is a zone with 4 colors, that applies that G cannot be colour with 3 colors.

B– There N are no zones with 4 colors, then G is to colour with 3

Re: a proof for consideration

colors.

the number of the triangles in this case, compared to the numbers of the vertex is polynomial: $t = (n-3) * 2$.

Two simple zones cannot thus have the same triangle the number of the zones is also polynomial.

Suppose that G is coloured by 4 colors, in this case a vertex (4) is adjacent at the three vertex (1, 2 and 3). If 1 is in Z_1 , 2 in Z_2 , and 3 in Z_3 ; since the vertex are coloured forcing then there is a relation enters to, the 3 zones form a zone made up.

Remain the case general, if the graph has 4-cliques then it could not be coloured by 3 colors.

Er, could you give an explicit example of your algorithm at work, where you use all cases mentioned? This isn't that clear.

My first question is: What if G has no triangles? The pentagon, for instance, has no triangles but requires 3 colors.

--- Christopher Heckman

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