Re: Range of the averages convex?

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- *From*: "TCL" <tlim1@xxxxxxxxxxx>
- *Date*: Mon, 09 Oct 2006 12:17:22 GMT

"The World Wide Wade" <waderameyxiii@xxxxxxxxxxxxxxxxxxxx> wrote in message
news:waderameyxiii−22DD27.13150008102006@xxxxxxxxxxxxxxxxxxxxxxxxxxx

> In article <iY9Wg.1802$ji5.603@trnddc04>,
> "johnson" <johnson246@xxxxxxxxxxx> wrote:
>
>> "johnson" <johnson246@xxxxxxxxxxx> wrote in message
>> news:hq9Wg.2204$e65.413@xxxxxxxxxxx
>>
>>> "johnson" <johnson246@xxxxxxxxxxx> wrote in message
>>> news:sm9Wg.2203$e65.1080@xxxxxxxxxxx
>>>
>>>> Let f be an essentially bounded Lebesgue
>>>> measurable function on
>>>> [0,1].
>>>> Consider the set
>>>> \int_E f d\mu
>>>> where \mu is the Lebesgue measure and E
>>>> are Lebesgue measurable
>>>> subsets
>>>> of [0,1].
>>>> Must f be convex?
>>>
>>> Correction:
>>>
>>> Let f be an essentially bounded Lebesgue measurable
>>> function on [0,1].
>>> Consider the set S=
>>> \int_E f d\mu
>>> where \mu is the Lebesgue measure and E are Lebesgue
>>> measurable subsets
>>> of
>>> [0,1].
>>> Must S be convex?

> > Sorry. I meant 1/mu(E) times \int_E f d\mu
> > for E such that mu(E)>0.
>
> If f is real valued, let a = ess inf f, b = ess sup f. Let eps > 0. By the Lebesgue differentiation theorem, we can find intervals I and J, of equal length h, such that 1/m(I)*int_I f < a + eps, 1/m(J)*int_J f > b − eps. The function x −> (1/h)int_(x,x+h) f is continuous, hence its range is an interval, which must contain [a+eps, b−eps]. This shows S contains (a,b), and since S is contained in [a,b], S is an interval, hence is convex.

What if f is complex valued? The above argument does not seem to work.

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