

probability of a dummy pops up

Source: <http://sci.tech-archive.net/Archive/sci.math/2006-10/msg05510.html>

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 - *Date:* 19 Oct 2006 23:20:15 -0700
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Could you please think about this?? This may come from engineering problem

Suppose there is a dummy-pops-up, and there is a mechanism controlling how often a dummy pops up.

The mechanism is as follow:

1. There is a switch, with "up" and "down" positions, to determine whether a dummy pops up or not. Every time the switch is triggered, the switch will either point to "up" or "down". The switch is triggered n times every 3 seconds (3 seconds, not 1). Moreover, for calculation purpose, $1 \leq n \leq 5$
2. If the switch points at "up", the dummy will pop up, if it points at "down", the dummy will not pop up.
3. Once the switch points at "up", it'll immediately return to the "down" position and will not point at "up" in the next 3 seconds. In other words, the dummy is not able to pop up more than once in any 3 second interval.
4. If the switch has pointed at "down" for more than 3 seconds, then whenever the switch is triggered, there is a 20% chance that the switch will point at "up".

Example: Suppose at $t=0$ the dummy pops up, and the switch is triggered every 0.7 second. Then at $t=0.7, 1.4, 2.1, 2.8$, the dummy will definitely not popped up. At $t=3.5$, there is 0.2 chance that the dummy will pop up. If the dummy doesn't pop up, then at $t=4.2$ there is 0.2 chance that it'll pop up.

Question: Suppose the switch is triggered x times. Denote the number that the dummy pops up by $k(x)$. What is $k(x)/x$ if x tends to infinity?

Hint: If the switch is triggered once every 4 seconds, then obviously the probability that the dummy pops up is 20%. However, if the switch is triggered more often, the mechanism will lower the probability that the dummy pops up.

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What I think is:

Let

1. The switch is triggered x times
2. The switch is triggered n times every 3 seconds
3. $Q(x,n)$ be the probability of the dummy pops up during the period that the switch is triggered x times

$$Q(x,n) = 0.8Q(x-1,n) + 0.2Q(x-n,n)$$

where $Q(x,n)=0$ if $x < n$

$Q(x,n)=1$ if $x=n$

To solve $Q(x,n)$ and then take limit $x \rightarrow \infty$!

Am I correct??

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