

A question on algebraic circle fitting

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Given a finite number of points in the plane,
(x_1, y_1), (x_2, y_2), ..., (x_n, y_n),
the "algebraic way" to fit a circle to these points
is to minimize the sum

$$\sum (x_i^2 + y_i^2 + 2 D x_i + 2 E y_i + F)^2, i = 1..n$$

over the variables D,E,F.

Is it correct that the resulting minimizers
will satisfy the condition

$$D^2 + E^2 \geq F ?$$

Under what conditions will minimizers D,E,F exist? –
A necessary condition is that not all points (x_i, y_i) lie on
a common line in case $n \geq 3$. Is this condition also sufficient,
or what other conditions are there?

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