

Re: Cardinals as Equivalence class?

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zuhair wrote:

Hi All,

In a previous Post, Moe Blee showed a proof that Frege's definition of Cardinality is not in ZFC.

from what I remember his proof was like that.

Let c a set of all injectable sets to set x . where x is not empty

now let $z \in x$.

then let Any $v: v \notin x$ replace z in x , then the resulting set is in c

$y = (x \setminus \{z\}) \cup \{v\} \rightarrow y \in c$.

from this Moe Blee deduces that Uc is the set of all sets.

Therefore c is not a set in ZFC.

My reply: I don't know till now how Uc is the set of all sets?

We have v is in Uc , and v was arbitrary, except that it's not in x .
It's easy to prove that everything in x is also in Uc . So every set is a member of Uc .

there is an intermediate step that I don't know in this proof.

Second: I personally have something which supports Moe Blee idea though from another angle.

Since v is any set that is not in x , and since c is a set that is not in x (axiom of regularity), then c itself can be v .

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ie c itself can replace z in x and thus can be a member of x , then c is a member of a member of c . which clearly violates the axiom of regularity.

Yes, that's another way.

ie instead of writing $\forall x (x \in c \rightarrow x \in c)$, since $x \in c$ then $c \in x$ then, we can take c itself.

$y = (x \setminus \{z\}) \cup \{c\} \rightarrow y \in c$ which violates the axiom of regularity, since c is a member of y .

so c is not a set in ZFC, But this can be corrected if we say that c is a PROPER CLASS

Yes. That's right. Frege's definition works in NBG, which has proper classes.

since proper classes cannot be members of other sets, ie do not have supersets.

I don't know whether if we assume that c is a proper class will also work in the same way on Moe Blee's proof, since $\cup c$ (I think this is the set union of c , ie the superset of c) do not exist for c if c is a proper class.

Yes, it does. As before, it turns out that $\cup c$ is the universe.

so an Equivalence class of sets, under equivalence relation "bijection" is in reality a proper class and it would be better written as.

Cardinal \leftrightarrow Equivalence proper class of sets, under equivalence relation "bijection".

Similar thing applies to the definition of ordinals as equivalence classes, it should be like below:

Ordinal \leftrightarrow Equivalence proper class of sets, under equivalence relation "order isomorphism".

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Yes, these definitions work fine in NBG.

The only trouble is we would like to have sets of ordinals and cardinals, and proper classes can't be members of sets.

There are ways around this. Von Neumann showed how to define the ordinals in ZF. You define them as transitive connected sets. The idea is that each ordinal is the set of all preceding ordinals. Then for cardinals you can use Scott's trick. There is a way to define the rank of a set, and the rank of a set is always an ordinal. In each case the rank of a set is the first ordinal after all the ranks of the members of the set. Then you can define the cardinality of a set to be the set of all sets equipollent to that set of smallest possible rank. You can prove that's a set. Or, if you're working in ZFC rather than ZF, just define a cardinal to be an ordinal not equipollent to any smaller ordinal.

Zuhair