Re: The cosine of a matrix

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In article <ehlbla$4a9$00$1@xxxxxxxxxxxxxxxxx> Gottfried Helms <helms@xxxxxxxxxxxxx> writes:

> Am 24.10.2006 17:20 schrieb C6L1V@xxxxxxx:
>
> > schoenfeld.one@xxxxxxxxx wrote:
> >
> > > In this article we shall derive an explicit formula for the
> > > cosine of a
> > > matrix.
> >
> > All if this is old hat and well−known; see, eg., Gantmacher, or
> > Lancaster.
> >
> > I have a related question, though: is it true that for a matrix A we
> > have
> > (sin(A))^2 + (cos(A))^2 = I? (I = identity matrix)
>
> I think, this is simple:
> it sums the (diagonal matrix of) eigenvalues to 1 and leads
> by
> E *cos(D)²*E^−1 + E *sin(D)²*E^−1

If the matrix is not defective (i.e. is diagonisable). Off−hand I
do not know whether it is also valid for defective matrices (I
suspect not).

The original poster ignores the possibility where the two eigenvalues are
equal. His formulas do not even work for the identity matrix.

On the other hand, if a matrix is diagonisable, it can be written as
M = U.D.V
where D is a diagonal matrix and where V.U = I. It is easy to show that
cos(M) = U.cos(D).V
and from that that cos(M)^2 + sin(M)^2 = I.

Somebody else questioned the usability. It has its uses in sets of

differential equations.

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