Re: An uncountable countable set

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- *From*: Tony Orlow <tony@xxxxxxxxxxxx>
- *Date*: Fri, 27 Oct 2006 11:47:20 −0400

RLG wrote:

> "Tony Orlow" <tony@xxxxxxxxxxxx> wrote in message
> news:4540f9d0@xxxxxxxxxxxxxxxxxxxxx
>
> > This is very simple. Everything that occurs is either an addition of ten balls or a removal of 1, and occurs a finite amount of time before noon. At the time of each event, balls remain. At noon, no balls are inserted or removed. The vase can only become empty through the removal of balls, so if no balls are removed, the vase cannot become empty at noon. It was not empty before noon, therefore it is not empty at noon. Nothing can happen at noon, since that would involve a ball n such that 1/n=0.
>
> Tony, I think your confusion results from imagining the balls without any labels. In this case at 1 minute before noon 10 balls are inserted into the vase, at 1/2 minute before noon 9 balls are inserted into the vase, at 1/4 minute before noon, 9 more balls are inserted into the vase and, in general, at (1/2)^n minutes before noon 9 balls are inserted into the vase. So you are saying that the number of vase balls at noon is:
>
> 10 + 9 + 9 + 9 + 9 + ... = Infinite.

Yes, but I don't consider that confusion. If the problem is solvable without the labels, then the labels don't matter.

> Or, since one ball is removed each time ten more are added, we should write:
>
> 10 + (10−1) + (10−1) + (10−1) + (10−1) + ... = Infinite.
>
> Now, this divergent series is conditionally convergent. That means we can make the sum equal any value we like depending on how the terms are arranged. So if we choose 0 for the

> sum that is perfectly valid:
>
> 10 + (10−1) + (10−1) + (10−1) + (10−1) + ... = 0.

No, we went through this in another thread. The only way to get a sum of 0 is by rearranging the terms and grouping so you have ten −1's for every +10. But, the sequence of events is specified NOT to be in that order. No ball can be removed without having ten inserted immediately before. So, despite the silly games that mathematicians may play with "conditionally convergent" series, none of that applies to the ball and vase problem as stated. Does that sound confused to you?

> In this case there are no balls in the vase at noon. Without labels on the balls there is no criterion by which to select what the sum should be and the end state of the supertask is undefined. As I noted in an earlier post, if some of the balls are labeled with numbers that are not naturals, for example transfinite ordinal numbers, we can choose "Infinite" for the sum if the circumstances require it.

No, the order cannot be rearranged. For each iteration you have a net addition of nine balls. You cannot remove a ball without adding ten more. This is clearly the divergent sum(n=1−>oo: 9).

> Consider the following problem:
>
> Tony has a two gallon bucket and his job is to ensure that the amount of water in the bucket during the nth day is 1+sin(n) gallons. Since Tony's job never ends he will always be making daily changes in the bucket's water content and we have a full mathematical description of Tony's job. There is no problem with this. But if we changed Tony's job so that it had an end, say at noon, and the bucket had to contain 1+sin(n) gallons at (1/2)^n minutes before noon then we do not have a full description of Tony's activities. It is a mistake to assume the bucket's water content at noon is a function of its pre−noon state. At noon Tony puts whatever amount of water he wants into the bucket.
>
> −R

Now you are talking about a series which diverges due to oscillation, more or less. At noon the bucket would have to be filling and emptying infinitely quickly. So what? Clearly, at noon, it has somewhere between 0 and 2 gallons of water, but no specific quantity. That's a different problem.
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