

# Re: Cantor Confusion

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  - *Date:* 3 Nov 2006 06:15:46 -0800
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MoeBlee schrieb:

0  
1 2  
3 4 5  
6 7 8 9

The entries surpass every finite entry.  
Nevertheless you call all of  
them finite.

I don't know what you're trying to say.

Because you did not read what I wrote. I defined it above: "better say  
finite sequences or numbers or entries"

No, I read it over a few times. When I say I don't understand  
something, you can take me at my word that I mean just that – I read it  
a few times, thought about it, and don't understand it. Thus, you can  
save yourself the wasted typing of saying false things such as that I  
didn't read what you wrote.

Entries are 1 2 and 3 4 5 and so on. The numbers written in a line.

I don't know what you mean by entries SURPASSING every finite entries.  
What entries surpass which other entries? What does 'surpass' mean? If  
you give me ordinary discourse, then I'll have a better chance of

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understanding you, just as I defined each of my terms, 'sequence', 'entry', etc. in my own remarks.

The digits of the numbers written down in your list (above) are not bounded by a finite number.

Maybe, if you say so. But  $\omega$  is not the maximum of all finite sequences.

Yes, since  $\omega$  is not a sequence at all, let alone being a finite sequence, let alone being the maximum of all finite sequences.

Therefore the width of the list is less than  $\omega$ .

My argument does not mention 'width of the list'. If YOU want to refer to 'width of the list', then YOU need to define it. And that means first proving that there exists a unique object that meets the description.

The width of the list is the number of digits of the number with most digits. As such a number does not exist, the width is the supremum, namely  $\omega$ .

The diagonal of the list is infinite.

That is your assertion. But obviously the diagonal elements are simultaneously elements of the entries.

No, we trivially PROVE the diagonal sequence is infinite.

You may also prove that the maximum of numbers less than 5 is 5. Nevertheless it is false.

No, I can't prove that.

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The diagonal of a list of sequences with less than 5 terms is less than 5.

The diagonal of a list of sequences with less than omega terms is less than omega.

This simple truth should convince you that ZFC is not acceptable.

You claim it is a simple truth without proving it. And your claim is not even compatible with the simple intuitive picture that uses ellipses. So not only do you not have a mathematical proof of your claim, you don't have an intuitive explanation, except an argument by ANALOGY in which you analogize between the finite and infinite, only assuming, as a form of question begging, that what holds for a finite sequence must hold for an infinite sequence.

I did not introduce a number omega which is larger than every natural number.

But IF such a number is introduced, THEN we should be allowed to use the inequality  $\omega > n$  for every natural number  $n$ , i.e. for the  $n$  digits of the  $n$ -th list entry.

The diagonal elements are simultaneously elements of the entries.  
Therefore the diagonal elements cannot sum up to a number which is larger than any natural number unless also the elements of list entries sum up to a number which is larger than any natural.

In my example, I said nothing about summing up. And I said nothing about anything in  $S$  being larger than any natural number.

You said the domain is omega. You said "we trivially PROVE the diagonal sequence is infinite". omega is larger than any natural number. "Infinite" means "larger than any natural number".

The common definition of 'is infinite' I use is:

$x$  is infinite  $\leftrightarrow \neg \exists n (n \text{ is a natural number} \ \& \ x \text{ is equinumerous with } n)$

which in turn reduces to:

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$x$  is infinite  $\leftrightarrow \sim \exists f (n \text{ is a natural number} \ \& \ f \text{ is a bijection between } x \text{ and } n)$

No mention there of "larger".

So you do not mean that  $\omega > n$  holds for every  $n \in \mathbb{N}$ ? Then we have no dissent.

Or put it so: Every segment of the diagonal is covered by an entry.

Which 'entries'?

There is no segment which is not covered.

What is the initial segment  $\{ \langle 0 \ 2 \rangle \}$ , of the diagonal, covered by? And what does it matter?

If all entries are finite,

Yes, all entries of  $S$  are finite sequences.

Without a maximum.

Yes, if you mean that there is no entry that has a greater length (notice, by the way, that 'greater' here is just the usual 'greater than' relation among natural numbers; i.e., finite) than all other entries.

Without a sequence of infinite length.

Correct. No entry of  $S$  has infinite length.

But the list has a diagonal which, if mapped on a line, has infinite length.

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The diagonal is an infinite sequence. So the diagonal is longer than any of the finite sequences. But the diagonal consists of elements of the finite sequences. So it cannot be longer than the maximum of the finite sequences.

There is NO "maximum of the finite sequences". If you want to use "the maximum of the finite sequences" in your argument, then you need to prove that there exists an object that meets that description.

If this maximum does not exist, you cannot take the supremum  $\omega$  for it, because the supremum is not a member of the sequences and does not supply elements of the diagonal.

I said nothing about taking a supremum.

Please address the proof I gave; not a strawman of my proof.

$\Omega$  is used in set theory. It is the supremum of the sequence of

If your next response is just more strawman and use of descriptions not proven to properly refer, then I may very well just note that rather than waste my time yet again explaining your own errors to you.

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For reference, here is my proof:

A sequence is a function such that the domain of the function is an ordinal. A finite sequence has a natural number as its domain. A denumerable (countably infinite) sequence has  $\omega$  as its domain. An uncountable sequence has an uncountable ordinal as its domain.

The entries in a sequence are members of the range of the sequence. Each entry is indexed by a member of the domain. The elements of the sequence are ordered pairs of the form  $\langle x, y \rangle$  where  $x$  is a member of the

domain (which is an ordinal) and  $y$  is a member of the range of the sequence (so the  $y$ 's are the entries).

$\text{dom}(S) =$  the domain of  $S$ .

$\text{length}(S) = \text{dom}(S)$ .

In my example  $S$  is the denumerable sequence recursively defined as follows:

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$$S(0) = \{ \langle 0 \ 0 \rangle \}$$

$S(n+1)$  = the unique finite sequence  $f$  such that  $\text{length}(f) = \text{length}(S(n))+1$  and such that  $f(0) = S(n)(\max(\text{dom}(S(n)))) + 1$

So  $S$  is a denumerable sequence such that each entry of  $S$  is a finite sequence.

The diagonal of  $S$  = the unique denumerable sequence  $D$  such that, for all  $n$  in  $\omega$ ,  $D(n) = S(n)(n)$ .

The diagonal of  $S$  is a denumerable sequence.

Therefore, there exists a denumerable sequence  $S$  of finite sequences such that the diagonal of  $S$  is denumerable.

Now map it on a line. It is longer than any line entry. But all line entries which can exist are already there. Hence the mapping of the diagonal cannot exist.

Regards, WM

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