

Re: Cantor Confusion

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- *From:* "MoeBlee" <jazzmobe@xxxxxxxxxxx>
 - *Date:* 3 Nov 2006 11:59:48 -0800
-

mueckenh@xxxxxxxxxxxxxxxxxxxx wrote:

MoeBlee schrieb:

0
1 2
3 4 5
6 7 8 9

The entries surpass every
finite entry. Nevertheless
you call all of
them finite.

I don't know what you're trying to say.

Because you did not read what I wrote. I defined it above:
"better say
finite sequences or numbers or entries"

No, I read it over a few times. When I say I don't understand something, you can take me at my word that I mean just that – I read it a few times, thought about it, and don't understand it. Thus, you can save yourself the wasted typing of saying false things such as that I didn't read what you wrote.

I don't know what you mean by entries SURPASSING every finite entries. What entries surpass which other entries? What does 'surpass' mean? If

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you give me ordinary discourse, then I'll have a better chance of understanding you, just as I defined each of my terms, 'sequence', 'entry', etc. in my own remarks.

The digits of the numbers written down in your list (above) are not bounded by a finite number.

S is a denumerable sequence of finite sequences. But, yes, of course, every natural number is a member of the range of exactly one of the finite sequences that are in the range of S. So, yes, of course, the union of the ranges of the finite sequences is unbounded.

So let's call that union, 'E' (which is, as I understand, what you would call the set of entries).

Maybe, if you say so. But ω is not the maximum of all finite sequences.

Yes, since ω is not a sequence at all, let alone being a finite sequence, let alone being the maximum of all finite sequences.

Therefore the width of the list is less than ω .

My argument does not mention 'width of the list'. If YOU want to refer to 'width of the list', then YOU need to define it. And that means first proving that there exists a unique object that meets the description.

The width of the list is the number of digits of the number with most digits. As such a number does not exist, the width is the supremum, namely ω .

First, my proof says nothing about digits or numbers of digits. My proof doesn't even require that we have proven the basis representation theorem.

Then, to make your definition precise, this is how I take it:

The width of S is the number of digits in the element of E that has more digits than any other element of E, if such an element of E exists, and otherwise the width of S is the supremum (by ordinal ordering) of the set of numbers of digits of elements of E.

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And, so, yes, under that definition, the width of S is omega.

The
diagonal of
the list is
infinite.

That is your assertion. But
obviously the diagonal
elements are
simultaneously elements of
the entries.

No, we trivially PROVE the diagonal
sequence is infinite.

You may also prove that the maximum of numbers less than
5 is 5.
Nevertheless it is false.

No, I can't prove that.

The diagonal of a list of sequences with less than 5 terms is
less than
5.
The diagonal of a list of sequences with less than omega
terms is less
than omega.

This simple truth should convince you that ZFC is not
acceptable.

You claim it is a simple truth without proving it. And your claim is
not even compatible with the simple intuitive picture that uses
ellipses. So not only do you not have a mathematical proof of your
claim, you don't have an intuitive explanation, except an argument by
ANALOGY in which you analogize between the finite and infinite, only
assuming, as a form of question begging, that what holds for a finite
sequence must hold for an infinite sequence.

I did not introduce a number omega which is larger than every natural
number.

But IF such a number is introduced, THEN we should be allowed to use

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the inequality $\omega > n$ for every natural number n , i.e. for the n digits of the n -th list entry.

If ' $>$ ' stands for the standard ordinal ordering, then, yes, of course, $\forall n (n \in \omega \rightarrow n < \omega)$.

But that doesn't entail that your argument by analogy has any merit whatsoever.

The diagonal elements are simultaneously elements of the entries.
Therefore the diagonal elements cannot sum up to a number which is larger than any natural number unless also the elements of list entries sum up to a number which is larger than any natural.

In my example, I said nothing about summing up. And I said nothing about anything in S being larger than any natural number.

You said the domain is ω . You said "we trivially PROVE the diagonal sequence is infinite". ω is larger than any natural number.
"Infinite" means "larger than any natural number".

The common definition of 'is infinite' I use is:

x is infinite $\leftrightarrow \neg \exists n (n \text{ is a natural number} \ \& \ x \text{ is equinumerous with } n)$

which in turn reduces to:

x is infinite $\leftrightarrow \neg \exists f (f \text{ is a bijection between } x \text{ and } n)$

No mention there of "larger".

So you do not mean that $\omega > n$ holds for every $n \in \mathbb{N}$? Then we have no dissent.

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No, of course, if ' $>$ ' stands for the standard ordinal ordering, then $\omega > n$ holds for every n in ω . But all I said was that my proof does not mention that. My proof does not require mentioning that. Just because my proof does not mention it does not mean that my proof contradicts it.

Or put it so: Every segment of the diagonal is covered by an entry.

Which 'entries'?

There is no segment which is not covered.

What is the initial segment $\{<0 2>\}$, of the diagonal, covered by? And what does it matter?

If all entries are finite,

Yes, all entries of S are finite sequences.

Without a maximum.

Yes, if you mean that there is no entry has a greater length (notice, by the way, that 'greater' here is just the usual 'greater than' relation among natural numbers; i.e., finite) than all other entries.

Without a sequence of infinite length.

Correct. No entry of S has infinite length.

But the list has a diagonal which, if mapped on a line, has infinite length.

First, my proof does not require mentioning mapping to lines (or "on a line"). Second, the diagonal is a sequence, so I don't know what you

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mean by mapping a sequence "on a line" (and I take it that you mean the real line or some segment of the real line), unless you mean an injection from the RANGE of the sequence into a line. But that is trivial to do, so I suspect that is not what you mean either. So maybe you have in mind some particular order preserving mapping. Or, perhaps what you're driving at is the fact that the diagonal is infinite. Yes, indeed, the diagonal is denumerable, which is what I proved, and that does not contradict anything about the real line or any mappings from any set to the set of real numbers or any order preserving mappings.

Look, I claimed that the diagonal is infinite. As I understand, you disputed that. So I proved that the diagonal is countably infinite. Now what is it that you are disputing? That the diagonal is infinite? That the diagonal is countable? I mean, you seem to be AGREEING with me that the diagonal is infinite? So what are you trying to argue now? That the infinitude or the countability of the diagonal shows a contradiction in set theory? If so, then please state the sentence P in the language of set theory such that you believe you have proofs in set theory of P and of $\sim P$.

If my proof is incorrect as a proof in Z, then either I've not used first order reasoning correctly in some step of my proof or I've used some premise that is not derivable with first order reasoning from the axioms of Z. But you don't point out anything like that. Instead, you raise considerations that are not in my proof. So I can only surmise that you think these considerations somehow contradict my proof, thus, since you have not refuted my proof, but have only shown what you think are contradictions between my proof and other considerations, then, if these considerations are part of Z, then Z is inconsistent (if the considerations are not part of Z, then, as I've said, of course I make no warranty that my proof is consistent with your personal, informal notions). So, if that is the case, again, what proof do you think you have, in set theory, of a sentence P & $\sim P$?. Moreover, IF set theory is inconsistent, then every sentence in the language of set theory is a theorem of set theory, so it would make no sense for you to claim of any sentence in set theory, including that the diagonal of S is denumerable, that it is not a theorem of set theory.

So, please say which step in my proof you believe is not justified by first order logic applied to the Z axioms, or please state what proof you think you have, in set theory, of a sentence P & $\sim P$.

The diagonal is an infinite sequence. So the diagonal is longer than any of the finite sequences. But the diagonal consists of elements of the finite sequences. So it cannot be longer than the maximum of the finite sequences.

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There is NO "maximum of the finite sequences". If you want to use "the maximum of the finite sequences" in your argument, then you need to prove that there exists an object that meets that description.

If this maximum does not exist, you cannot take the supremum ω for it, because the supremum is not a member of the sequences and does not supply elements of the diagonal.

I said nothing about taking a supremum.

Please address the proof I gave; not a strawman of my proof.

Ω is used in set theory. It is the supremum of the sequence of

I mentioned nothing about supremum. And the fact that ω is the supremum (by the standard ordinal ordering) of any particular set does not contradict anything in my proof.

If your next response is just more strawman and use of descriptions not proven to properly refer, then I may very well just note that rather than waste my time yet again explaining your own errors to you.

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For reference, here is my proof:

A sequence is a function such that the domain of the function is an ordinal. A finite sequence has a natural number as its domain. A denumerable (countably infinite) sequence has ω as its domain. An uncountable sequence has an uncountable ordinal as its domain.

The entries in a sequence are members of the range of the sequence. Each entry is indexed by a member of the domain. The elements of the sequence are ordered pairs of the form $\langle x, y \rangle$ where x is a member of the

domain (which is an ordinal) and y is a member of the range of the sequence (so the y 's are the entries).

$\text{dom}(S) = \text{the domain of } S.$

$\text{length}(S) = \text{dom}(S).$

In my example S is the denumerable sequence recursively defined as follows:

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$$S(0) = \{ \langle 0 \ 0 \rangle \}$$

$S(n+1)$ = the unique finite sequence f such that $\text{length}(f) = \text{length}(S(n))+1$ and such that $f(0) = S(n)(\max(\text{dom}(S(n)))) + 1$

So S is a denumerable sequence such that each entry of S is a finite sequence.

The diagonal of S = the unique denumerable sequence D such that, for all n in ω , $D(n) = S(n)(n)$.

The diagonal of S is a denumerable sequence.

Therefore, there exists a denumerable sequence S of finite sequences such that the diagonal of S is denumerable.

Now map it on a line. It is longer than any line entry.

The real line? What is your definition of 'length' (since you use 'longer than') of entries in the real line? And 'entry' on the real line is something yet again we need to define. But I suppose here that you just mean that the entries on the real line are the real numbers? So what is the 'length' you have in mind for real numbers? The length of certain sequences that represent those real numbers? But that wouldn't seem to go with the idea of mapping the diagonal "on a line" and comparing lengths of entries. So, you'll have to define your terms.

But all line entries which can exist are already there. Hence the mapping of the diagonal cannot exist.

Here, I'll repeat what I wrote earlier in this post, since, though somewhat lengthy, these are remarks that do bear repeating:

First, my proof does not require mentioning mapping to lines (or "on a line"). Second, the diagonal is a sequence, so I don't know what you mean by mapping a sequence "on a line" (and I take it that you mean the real line or some segment of the real line), unless you mean an injection from the RANGE of the sequence into a line. But that is trivial to do, so I suspect that is not what you mean either. So maybe you have in mind some particular order preserving mapping. Or, perhaps what you're driving at is the fact that the diagonal is infinite. Yes, indeed, the diagonal is denumerable, which is what I proved, and that does not contradict anything about the real line or any mappings from any set to the set of real numbers or any order preserving mappings.

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MoeBlee

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