Re: A simple question?

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- *From*: David Marcus <DavidMarcus@xxxxxxxxxxxxxx>
- *Date*: Fri, 3 Nov 2006 18:21:10 −0500

MoeBlee wrote:

> David Marcus wrote:
>
> > To me, if someone says a set is well−ordered, they mean whatever ordering relation the set has
>
> First, I stress that this is a matter of how we use informal language about set theory. So what is most important is for us to agree on our use so we understand one another, while the question of what is the "correct" (or at least the most common) informal use is secondary though important too.
>
> In accord with Suppes (here, 'e' for the epsilon membership symbol):
>
> R well orders S <−> R is connected in S & Ab(b is a nonempty subset of S −> Ex(xeb & Az(zeb −> ~ <z x>eR)))
>
> Then, personally, my use:
>
> R is a well ordering of S <−> R well orders S <−> S is well ordered by R

I'll go along with that.

> That's a two−place predicate.
>
> So:
>
> S is well ordered <−> ER R is a well ordering of S <−> ER R well orders S <−> ER S is well ordered by R
>
> That's a one−place predicate.

I don't think I've seen that use before.

> But what does your phrase, "whatever ordering relation the set has" mean? There can be many orderings of a set. If there is a well ordering of the set, then one of the orderings that the set "has" is a well ordering.

I mean whichever ordering relation has been specified on the set earlier in the exposition by the author or, if the author hasn't specified one, whichever ordering relation it would be natural to assume. For example, if I define a certain subset of the reals, I would expect the reader to assume the ordering inherited from the reals, unless I said otherwise.

>> (or whatever ordering relation we are
>> talking about) is a well−ordering.

> Okay, but then we have to have mentioned a particular ordering.

Exactly! Or, there has to be an obvious one from the context.

> If
> someone says "S well ordered", without mentioning a particular ordering, then I take that in the sense of "There is a a well ordering of S".

If someone says "S is well ordered" and it isn't clear to me which ordering of S he is saying is a well ordering, then I ask!

> We should not presume that the statement means "The standard ordering of S a well ordering", especially in a general context since, sets having standard orderings is an exception not a generality. And especially for the power set of omega, there is no notion whatsoever of a standard ordering anyway.

>> It doesn't mean that I can construct
>> a new ordering relation that is a well−ordering.

> What is a "new" ordering?

By "new", I meant one that I haven't mentioned before in the book or article or newsgroup post that I'm writing and one that isn't the natural ordering on the set which I might expect the reader to assume if I don't specify an ordering.

> All orderings of a set exist "simultaneously". And even a standard ordering of set is not something that exists in some kind of temporal or ontological priority over other orderings.

The temporal ordering is that of reading the book or article or newsgroup post.

> Standard orderings may be more naturally of concern or even more natural for us to construct as to our human interest in them, but they are "older" than other orderings only in the sense that we as humans may have studied them prior to other orderings. In the actual theory, they're not somehow "older" or more "primitive" than other orderings.
>
> > Do you see somewhere in Enderton where he uses "well−ordered" to mean a well−ordering exists?
>
> I don't know that the book has that, since I'm not looking for every mention in the book, but I do find right away:
>
> Page 196:
>
> "Well Ordering Theorem: For any set A, there is a well ordering of A. This theorem is often stated more informally: Any set can be well ordered."
>
> So, I take that to indicate that 'there is a well ordering of S' and 'S can be well ordered' are equivalent. And I take 'S can be well ordered' to be equivalent to 'S is well ordered', since surely "can" should not be taken to refer to human agency but rather I take it is as anthropomorphic only in a figurative sense. What "can" be be ordered IS ordered, whatever our human knowledge of the particular orderings may be.

That's why Enderton said it is informal: because the "can" is referring to the fact that I can say, "Let's use an ordering on A that is a well ordering". And, the Well Ordering Theorem says I am justified in saying this, if I want to.

To me, "S is well ordered" is different from "S can be well ordered". If someone says to me that "S is well ordered", and it isn't obvious to me which ordering they are using on S, then I would ask them what ordering they are referring to.

> Page 191:
>
> "A set is well ordered by epsilon iff [...]"
>
> That mentions not only the well ordering but the relation that is the well ordering. And I regard 'S is well ordered by R' to entail 'S is well ordered'.

It seems to me from your examples and my looking at the book that Enderton doesn't say "S is well ordered" without also saying what ordering he is referring to.

--
David Marcus
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