Re: A simple question?

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- *From*: David Marcus <DavidMarcus@xxxxxxxxxxxxxx>
- *Date*: Mon, 6 Nov 2006 00:42:45 −0500

MoeBlee wrote:

> David Marcus wrote:
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> > > > MoeBlee wrote:
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> > > > > David Marcus wrote:
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> > > > > > Do you see somewhere in Enderton where he uses "well−ordered" to mean a well−ordering exists?
> > > > >
> > > > > I don't know that the book has that, since I'm not looking for every mention in the book, but I do find right away:
> > > > >
> > > > > Page 196:
> > > > >
> > > > > "Well Ordering Theorem: For any set A, there is a well ordering of A.
> > > > > This theorem is often stated more informally: Any set can be well

ordered."

So, I take that to indicate
that 'there is a well ordering
of S' and 'S
can be well ordered' are
equivalent. And I take 'S can
be well ordered'
to be equivalent to 'S is well
ordered', since surely "can"
should not
be taken to refer to human
agency but rather I take it is
as
anthropomorphic only in a
figurative sense. What "can"
be be ordered IS
ordered, whatever our
human knowledge of the
particular orderings may
be.

That's why Enderton said it is informal:
because the "can" is referring
to the fact that I can say, "Let's use an
ordering on A that is a well
ordering". And, the Well Ordering Theorem
says I am justified in saying
this, if I want to.

To me, "S is well ordered" is different from
"S can be well ordered". If
someone says to me that "S is well ordered",
and it isn't obvious to me
which ordering they are using on S, then I
would ask them what ordering
they are referring to.

Okay, we do take this particular informal speaking
differently, then.

Page 191:

"A set is well ordered by
epsilon iff [...]"

That mentions not only the
well ordering but the

relation that is the
well ordering. And I regard
'S is well ordered by R' to
entail 'S is
well ordered'.

It seems to me from your examples and my
looking at the book that
Enderton doesn't say "S is well ordered"
without also saying what
ordering he is referring to.

No, I haven't looked through the book to see whether he uses
the plain
expression "S is well ordered" without mentioning a
particular well
ordering and, for all I know, he might agree with your way of
understanding such informal locutions.

But here's Suppes:

233:

"[...] either two well−ordered sets are similar or one is
similar to an
intitial segment of another".

I take that to mean that if you have two sets A1 and A2 and an ordering
R1 on A1 and an ordering R2 on A2 and R1 and R2 are well orderings, then
A1 and A2 are similar or one is similar to an initial segment of the
other.

Right. And there is some relation that is the ordering. But he doesn't
mention any specific ordering.

Yet, I'm starting to feel that your notion is closer to the usual way
the words are used and that my way of using the words is not ordinary.

First, I missed in Halmos that he mentioned that a partially ordered
set is a set WITH a partial ordering, so presumably, that would carry
over to a well ordered set so that when he talks about a well ordered
set, he's talking about a set with a well ordering (whereas, I was
(against the grain of ordinary use it is starting to seem to me)
advocating that we can talk about a well ordered set and just suppose
there is some well ordering but not supposing that we have a particular
well ordering in mind).

> Also, I notice that Bernays speaks of 'well orderable', which is
> another point against my argument.
>
> Also, I didn't mention that many writers refer to not just the set S
> but the pair <S R> where 'a well ordered set' is meant by these writers
> to mean a pair <S R> where R is a well ordering of S.

Yes, those are all good points.

> So, I wish not to persist to argue that my way of speaking is in accord
> with an ordinary way of speaking. (Nevertheless, of course, if I make
> explicit definitions such as "S is well ordered <−> ER R is a well
> ordering of S", then that is my prerogative, though I could not
> reasonably claim that my definition is ordinary if it is indeed not
> ordinary.)

Sure.

>>> And he mentions on that page:
>>>
>>> "[...] this theorem is an analogue for well−ordered sets [...]"
>>>
>>> 236:
>>>
>>> "[...] the fundamental theorem for well ordered sets [...]"
>>>
>>> And my take on "can" is in accord with Suppes:
>>>
>>> 242:
>>>
>>> "Every set can be well ordered; that is, for every set A there
>>> is a
>>> relation R such that R well−orders A."

>> Yes, I agreed with how you, Enderton, and Suppes interpret "can".

>>> And I really do NOT want to distinguish between the loose
>>> speaking
>>> "can" and the more ontologically exact "is".

>> But, I think Enderton and Suppes don't think "can" is the same as your
>> meaning for "is".

> I don't see why not. There's no modality of "can" in the formal theory. "can" is a loose way of speaking, which is okay, but I don't know of anything in IN the theory to distinguish 'can' from 'is'.

I think we can give a formal meaning (and I think Enderton would agree with this): If we say "S is a well ordered set", then we really mean "(S,R) is an ordered pair where S is a set, R is an ordering on S, and R is a well ordering". If we say "S can be well ordered", then we mean "S is a set and there exists R such that R is an ordering on S and R is a well ordering". So, with "is", R is unbound, but with "can" it is bound.

That's why people say that the axiom of choice means "Any set can be well ordered" rather than "Any set is well ordered". If I give you a set S, then it doesn't make sense to say S is an ordered pair (S,R), but it does make sense to say that I can find R such that (S,R) etc.

> > > Halmos:
> > >
> > > 70:
> > >
> > > "[...] on any well ordered set W [...]"
> > >
> > > He speaks of a set W being well ordered with no mention whatsoever of a
> > > particular ordering.
> >
> > I take that to mean that whatever he is about to say is true for any set W with an ordering R as long as R is a well ordering.
>
> Sure, but he's not mentioning that R any more than as I mention it when I use it as a bound variable for an existential quantifier in "S is well ordered <−> ER R is a well ordering of S" or "S is a well ordered set <−> ER R is a well ordering of S" so that R drops out COMPLETELY after it's done its job as a bound variable in a definition.

That's because whatever Halmos says has an implicit "for all R" wrapped around the outside of it, and the resulting statement is probably equivalent to your statement where the R is bound by your "there exists" quantifier.

> > > 70:
> > >
> > > "If W is a well ordered set [...]"

I.e., if W is a set and R is an ordering on W and R is a well ordering,
then ...

Even though I am conceding the general point to you, in fairness to
myself, in this example, you are blatantly injecting "R" when it is
simply not mentioned.

Yes. I believe it is part of the definition of the phrase "well ordered
set".

72:

"It is easily possible for a well ordered set to be similar to a
proper
subset [...]"

Again, speaking of a well ordered set without mentioning
any particular
well ordering.

73:

"[...] a well ordered set is never similar to one of its initial
segments [...]"

There he even mentions segments without saying anything
about the
ordering of which these are segments.

I take it to be the same ordering that is a well ordering of the entire
set.

Again, you assume "THE" well ordering [all caps added]. There usually
is not THE well ordering of the set, since there may be many well
orderings of the set. There is "the" well ordering only as soon as we
specify WHICH well ordering we are talking about so as to define "THE
well ordering such that [...]" So what he says applies to ANY well
ordering of the set.

But, Halmos's conclusion refers to the ordering, so you can't translate
what he has as "If S is a set and ER(R is a well ordering of S), then S
is never similar to one of its initial segments" because you need R to

give meaning to "similar" and to "segment". You have to do something like "If (S,R) is a well ordered set, then it is never similar to one of its initial segments."

> > > "[...] comparability theorem for well ordered sets [...]"
> > >
> > > "[...] if X and Y are well ordered sets [...]"
> > >
> > > And more on that page mentioning "well ordered sets" with no context
> > > whatsoever as to what the well ordering is.
>
> > It is the ordering that is also a well ordering.
>
> Again, you inject "the".

> > > And, in a theory in which every object is a set, such as ZF, I really
> > > do not see any point in distinguishing between "S is a well ordered
> > > set" and "S is well ordered".
>
> > Both of these phrases mean to me that we are talking about a set S, an ordering R on S, and R is a well ordering.
>
> But why can't we talk in greatest generality too? I can just say that there exists SOME well ordering of the set, without mentioning a particular well ordering. Then I can talk about such sets that have some well ordering on them.

But, isn't that what I'm doing? If I say I have a well ordered set (meaning I have a set and an ordering that is a well ordering) and then what I say about it doesn't use anything specific about the ordering (other than that it is a well ordering), then I am being general.

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David Marcus
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