Re: derivative of the matrix log

Source: http://sci.tech−archive.net/Archive/sci.math/2006−11/msg01744.html

---

- *From*: Stephen Montgomery−Smith <stephen@xxxxxxxxxxxxxxxxx>
- *Date*: Tue, 07 Nov 2006 00:00:44 GMT

Pouya D. Tafti wrote:

> Hello everyone,
>
> In one of the exercises for a noncredit course that I am attending it has been asked to show the following:
>
> d logA = A^−1 dA. (1)
>
> Here A is a nonsingular matrix and d supposedly denotes differentiation −− I am interpreting it as being with respect to a scalar parameter, but I may be wrong.

I don't think it is true unless A and dA commute. So I think the restrictions in your proof below are appropriate.

> Below is a description of my incomplete attempt at deriving (1). As you will see, I could certainly benefit from some advice.
>
> First of all, I remember (somewhat vaguely) from undergrad school that matrix exponentials may be defined by the absolutely convergent series
>
> I + sum_i>0 A^i / i!.
>
> I don't remember having seen matrix logarithms before, but if A is symmetric positive−definite with eigen−decomposition USU', then
>
> logA := U logS U'
>
> poses itself as an agreeable definition, as it satisfies
>
> exp logA = log expA = A.

For arbitrary A this definition may not work; but then again, regarding log as the inverse of exp one may generally write

$$A = I + \sum_{i>0} (\log A)^i / i!. \quad (2)$$

Now if for some matrix X, X and dX commute, one can show that

$$d(X^n) = n X^{(n-1)} dX.$$

Using this and assuming that (2) can be differentiated term−by−term (by some extension of the corresponding scalar result), I can derive (1), but only if logA and d(logA) commute. Is this extra condition really necessary for (1) to hold? If yes, how restrictive is it? If no, how can one prove (1) without it?

Thanks very much,

.