Re: Cantor set = irrationals

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- *Date*: 15 Nov 2006 15:27:16 −0800

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eugene wrote:

> Is it possible to construct fat Cantor set which
> consists only of irrational numbers ?

Yes. For lots of examples, see:

Duane Boes, Richard Darst, and Paul Erdos, "Fat, symmetric, irrational Cantor sets", American Mathematical Monthly 88 #5 (May 1981), 340−341.

For another way to get lots of examples, see the following sci.math post:

http://groups.google.com/group/sci.math/msg/ce542e3d90896bf1

Finally, I'm pretty sure the following is true, but I don't think anyone has published a proof yet:

Let 0 <= d < 1 and let C(d) be the collection of all closed subsets of [0,1] with measure >= d. [Exclude the empty set from this collection if d = 0.] Then C(d) is a complete metric space under the Hausdorff metric. (This is known.) I believe there exists a co−meager subset of C(d) each of whose members is a perfect set of measure d containing no rational points.

I know this is true for d = 0. Indeed, for d = 0 we can actually have each of the Cantor sets being an algebraically independent set of numbers. I also know this is true for 0 < d < 1, if we disregard the rational points requirement.

Dave L. Renfro

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