Re: (Very Off Topic): Need Help With Linear Maps Question

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[Dave L. Renfro]

> > Problem on test: Prove that f(x) = 2x − 1 is a 1−to−1 function.
> >
> > Actual student answer: The function takes 1 to 1, and therefore
> > it's a 1−to−1 function.

[David T. Ashley]

> Just out of curiousity (I'm not a mathematician), what is a valid
> proof that this is 1:1?

Start with the definition. Stop there too :−)

"f is 1−to−1" means:

f(x) = f(y) implies x = y

or, equivalently,

x =/= y implies f(x) =/= f(y)

(read "=/=" as "does not equal"). The first version is usually easiest to
use in a proof. In this case,

f(x) = f(y) implies [plugging in f(z) = 2z−1]
2x−1 = 2y−1 implies [adding 1 to both sides]
2x = 2y implies [dividing both sides by 2]
x = y

So f satisfies the definition of 1−to−1.

> I'm tempted to say that if g(x) is the inverse defined as (x+1)/2,
> then for any real number x you get:

> g(f(x)) = ((2x−1) + 1)/2 = x, i.e. you always get the same result back.
>
> This seems to prove 1:1, since if it were not 1:1 and both f() and g() are functions there would be some "non−reversible" value for the argument.

Showing that the inverse function is well−defined works too, since f is one−to−one if and only if f has an inverse defined on f's codomain. As above, it's usually easier to work directly with the definition.

> But how would a mathematician do this proof? (I'm just a hack.)

Ah, a /mathematician/ would say "it's obvious" and move on to the next post ;−)

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