Re: Cantor Confusion

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- *From*: Lester Zick <dontbother@xxxxxxxxxxx>
- *Date*: Fri, 24 Nov 2006 11:23:28 −0700

On Fri, 24 Nov 2006 01:10:39 GMT, "Dik T. Winter" <Dik.Winter@xxxxxx>
wrote:

> In article <34i6m255sfv5upv6nenqm6menle2gp8l8b@xxxxxxx> Lester Zick <dontbother@xxxxxxxxxxx> writes:
>
>> On Tue, 21 Nov 2006 03:04:42 GMT, "Dik T. Winter" <Dik.Winter@xxxxxx>
>> wrote:
>>
>>> In article <8m8pl2pj1icbeven2hq7rp4hq1rufqh1u2@xxxxxxx> Lester Zick <dontbother@xxxxxxxxxxx> writes:
>>> ...
>>>
>>>> Why are square circles unimaginable?
>>>
>>> They are not. With the Manhattan measure of the plane, each circle is
>>> a square.
>>
>> Then what is a square?
>
> Pray, tell me. I would say that a straight line in the Euclidean plane
> is a line of the form $ax + by = c$. With the standard formulas for angles
> it is easy enough to get rectangles. And I would say that a rectangle is
> a square when the sides have equal length (this is the point where the
> measure creeps in). So we have a rectangle enclosed by the lines:
> $x + y = 1$
> $x - y = 1$
> $-x + y = 1$
> $-x - y = 1$
> Now define the Manhattan measure:
> $d((x1,y1), (x2,y2)) = ||x1 - x2| + |y1 - y2||$
> and we see easily enough that the figure enclosed by the lines above is

> a square with sides with length 2.
>
> A circle is a figure where each point has the same distance to a common centre, and it is also easy to show that the points on the boundary of that square have the same distance to the origin: 0.

Well for one thing points equidistant from any point define a sphere not a circle unless one assumes "on a plane" when the Euclidean plane isn't defined to begin with. But my question was directed not at the definition of a circle or square on a Euclidean plane but at the definition of a square with the Manhattan measure. It looks to me that you've just defined a square with the Manhattan metric with the properties of a circle in Euclidean plane metric. What's the point of that if you don't define a square with different properties in the Manhattan metric?

In other words we have two different figures defined in the Euclidean metric, one as a curve and one with straight lines. Now I'm not trying to quibble over the modern math definition of either figure at the moment, just trying to point out that you have certain characteristics and properties defined in the Euclidean metric and then apparently claim that if you use some other metric and don't use the Euclidean metric the two figures are the same.

I suppose I should ask instead are there any Euclidean curves in the Manhattan metric? If not the issue is moot. But my primary concern is that I see people all the time defining lines, figures, etc. with the Euclidean metric then going on to do non Euclidean mathematics with them. I mean Euclidean definitions require the Euclidean metric. And if you want to use some other metric you need some other definition using that metric. In which case my original question should be rephrased "why are square circles unimaginable in the plane Euclidean metric"?

For example is it possible to define plane squares with non Euclidean metrics? And can we define right angles without the parallel postulate people simply ellide when operating with non Euclidean geometries? No.

Then people complain that what they do gives the same answers. But that's only because they're using the same words yet asking different questions, with concepts defined in the Euclidean metric but operated on in some non Euclidean metric but not defined in that metric. When I ask a question such as "why are square circles unimaginable" the defining metric for "squares" and "circles" is Euclidean and not Manhattan and I don't expect answers that if we look at Euclidean figures through some other metric we'll find they are imaginable.

~v~~

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