

# $Q(a^{1/n})$

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Let  $K = Q(a^{1/n})$ , where  $a \in Q$  and suppose  $[K: Q] = n$ . Let  $E$  be any subfield of  $K$  and let  $[E: Q] = d$ . Prove that  $E = Q(a^{1/d})$ .

Could you please help me to figure out with this problem.

What i've done : denote  $\alpha = a^{1/n}$ . Then consider the norm  $N_{K/E}(\alpha) = \alpha^{n/d} \cdot \text{product of } n\text{'th roots of unity} = \alpha^{n/d} \cdot \zeta^{ki}$

Then  $b \in E$  and  $b^d = a \cdot \zeta^{ki}$ . Now i don't know how to proceed, i'm not sure whether  $\zeta^{ki}$  is actually a  $d$ 'th root of unity, if it were, then we could find a solution of  $x^d - a$  in  $E$  and that would imply that  $Q(a^{1/d}) \in E$ .

Thanks.

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