Re: Cantor Confusion

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- *From*: Lester Zick <dontbother@xxxxxxxxxxx>
- *Date*: Sat, 25 Nov 2006 11:56:57 −0700

On Sat, 25 Nov 2006 03:12:20 GMT, "Dik T. Winter" <Dik.Winter@xxxxxx>
wrote:

> In article <lcbem29apkc14rfr1m1jtqaqj2anh1e41l@xxxxxxx> Lester Zick
> <dontbother@xxxxxxxxxxx> writes:
>
>> On Fri, 24 Nov 2006 01:10:39 GMT, "Dik T. Winter"
>> <Dik.Winter@xxxxxx>
>> wrote:
>
> ...
>
>>> Pray, tell me. I would say that a straight line in the Euclidean
>>> plane
>>> is a line of the form ax + by = c. With the standard formulas
>>> for angles
>>> it is easy enough to get rectangles. And I would say that a
>>> rectangle is
>>> a square when the sides have equal length (this is the point
>>> where the
>>> measure creeps in). So we have a rectangle enclosed by the
>>> lines:
>>> x + y = 1
>>> x − y = 1
>>> − x + y = 1
>>> − x − y = 1
>>> Now define the Manhattan measure:
>>> d((x1,y1), (x2,y2)) = ||x1 − x2| + |y1 − y2||
>>> and we see easily enough that the figure enclosed by the lines
>>> above is
>>> a square with sides with length 2.
>>>
>>> A circle is a figure where each point has the same distance to
>>> a common
>>> centre, and it is also easy to show that the points on the
>>> boundary of
>>> that square have the same distance to the origin: 0.

> > Well for one thing points equidistant from any point define a sphere not a circle unless one assumes "on a plane" when the Euclidean plane isn't defined to begin with. But my question was directed not at the definition of a circle or square on a Euclidean plane but at the definition of a square with the Manhattan measure.

> You completely misunderstand what I wrote. I *start* with an Euclidean plane without measure (i.e. distance function). With that we can at most define a rectangle. Neither a square, nor a circle.

Actually I think I understand you very well. How is it exactly you start with a Euclidean plane? The fact that you start without a distance measure doesn't allow you to start with a plane Euclidean or otherwise. If you begin by assuming this you wind up by assuming that and pretty soon you find yourself assuming what you were supposed to demonstrate in the first place, that square circles are conceivable.

> > In other words we have two different figures defined in the Euclidean metric, one as a curve and one with straight lines. Now I'm not trying to quibble over the modern math definition of either figure at the moment, just trying to point out that you have certain characteristics and properties defined in the Euclidean metric and then apparently claim that if you use some other metric and don't use the Euclidean metric the two figures are the same.

> I do not use Euclidean metric at all. Where, above, do I use Euclidean metric?

I suspect we're using the phrase "Euclidean metric" in different ways. When I use the term I'm referring not just to measures of distance but to all definitive characteristics which go into definitions of such things as dimensionality and geometric figures in addition to distance measures. For example I see no definition of yours for "plane" which I think would be impossible to define without a Euclidean metric. On the other hand if all you're describing are variable measures of distance then you'd have to explain how you obtain those measures without an underlying Euclidean metric and corresponding assumptions. You can't just assume them as modern mathematikers are wont to do.

> > For example is it possible to define plane squares with non Euclidean metrics?

> Of course.

Well then let's see some definitions for planes, circles, and squares which don't explicitly or implicitly rely on Euclidean assumptions.

> > And can we define right angles without the parallel postulate people simply ellide when operating with non Euclidean geometries? No.

> I think you can. But I used Euclidean geometry above.

But the problem here is what kind of definition for squares doesn't rely on straight lines, right angles, planes, and so on?

> > When I
> > ask a question such as "why are square circles unimaginable" the defining metric for "squares" and "circles" is Euclidean and not Manhattan and I don't expect answers that if we look at Euclidean figures through some other metric we'll find they are imaginable.

> In that case you should use better formulations in your questions.

And you should use better formulations in your answers.

> And
> when I follow−up to point out that in the Manhattan measure all circles are squares (but not the other way around) you should state that your formulation was insufficient.

Except that your answer relies on non Euclidean assumptions that circles are squares. If I ask why one sided quadrangles are unimaginable and you reply that they aren't if you start counting from four would you consider your answer responsive to the question asked? All you're doing is answering a question that wasn't asked in terms employed by the original question. I can make up private definitions just like everyone else does but that doesn't make definitions true.

In fact my personal favorite private definition for distance metrics is one I made up for the real number line which runs 1, 2, e, 3, pi, 4, 5, . . but I don't try to pretend that when I'm trying to analyze real numbers that that is a true definition.

Besides as far as I can tell you still haven't answered my question as to whether there are any curves at all with the Manhattan measure. In fact I can't even find it reprinted above.

~v~~
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