Re: Galileo's Paradox

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- *From*: Six Letters
- *Date*: Wed, 29 Nov 2006 12:02:32 +0000

On Tue, 28 Nov 2006 15:14:22 +0000 (UTC), stephen@xxxxxxxxxx wrote:

> Six wrote:
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> > On Mon, 27 Nov 2006 02:21:33 +0000 (UTC), stephen@xxxxxxxxxx wrote:
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> > > Six wrote:
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> > > > On Fri, 24 Nov 2006 18:26:37 +0000 (UTC), stephen@xxxxxxxxxx wrote:
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> > > > > Six wrote:
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> > > > > > On Fri, 24 Nov 2006 16:04:12 +0000 (UTC), stephen@xxxxxxxxxx wrote:
> > > > > >
> > > > > > > Six wrote:
> > > > > > >
> > > > > > > <snip>
> > > > > > >
> > > > > > > > I want to suggest there

are only two sensible ways to resolve the paradox:

1) So−called denumerable sets may be of different size.

2) It makes no sense to compare infinite sets for size, neither to say one is bigger than the other, nor to say one is the

same size as another. The infinite is just infinite.

My line of thought is that the 1:1C is a sacred cow. That there is no extension from the finite case.

What do you mean by that? The one−to−one correspondence works perfectly in the finite case.

That is the entire idea behind counting. Given any two finite sets, such as {q, x, z, r} and {#, %, * @}, there exists a one−to−one correspondence between them if and only if they have the same number of elements. This is the idea that let humans count sheep using rocks long before they had names for the numbers.

I love this quaint, homely picture of the origin of arithmetic. I am sure that evolutionary arithmetic will soon be taught in universities, if it is not already. Disregarding the anthropology, however, you have said absolutely nothing about whether !:!C is adequate for the infinite case.

I was addressing your claim that there was "no extension from the
finite case". In the finite case, two sets have the same number

of elements if and only if
there exists a one to one
correspondence
between them. This very
simple idea has been
extended to the
infinite case.

OK. The idea of a 1:1 corresondence is
indeed a simple idea. The idea of
infinity is not.

That depends on what 'idea of infinity' of you are talking
about.
The mathematical definition of 'infinite' is as simple as the
idea of a 1:1 correspondence.

The mathematical definition of infinity may be simple, but is it
unproblematic? It seems to me that infinity is a sublte and difficult
concept.

What concept of infinity? Note, I said 'infinite', not 'infinity'.
You have been talking about Cantor and one−to−one correspondences,
so you have been talking about set theory. The word 'infinity'
is generally not used in set theory. It has no formal definition.
'infinite' is used to describe sets, and it has a very simple
definition.

I'm talking about mathematical meaning. Specifically I'm talking
about "How many?", more or less etc..

And that we are entitled to ask how well the simple mathematical
defintion captures what we mean by it, not necessarily in all its wilder
philosphical nuances, but what we mean by it mathematically, or if you
like, proto− mathematically.

A set is infinite if there exists a bijection between the set and
a proper subset of itself. That is what mathematicians mean when
they say a set is infinite. There are other equivalent definitions.

I know already.

There is no point in dragging
philosophical baggage into a mathematical discussion.

In my opinion the philsosopy is already there, and it impoverishes
mathematics to pretend otherwise.

Do you have the same problem with prime numbers? Or even numbers?
The words 'prime' and 'even' have meanings outside of mathematics.
Do you feel obligated to drag those meanings into a discussion
of prime or even numbers?

See above

<snip>

?? How do I know what the
missing elements are?

The one−to−one
correspondence idea is nice
because it works for any
two sets. The idea you are
looking at only works if one
set
is a subset of the other.

Yes, to set up the paradox we need to
compare two sets for which
there is a 1:1C and one is a subset of the
other. It isn't a question of
what works. It's a question of how the
paradox is to be resolved.

Thanks, Six Letters

There is no need to resolve the paradox. There exists a
one−to−correspondence between the natural numbers and the

perfect squares. The perfect squares are also a proper
subset of the natural numbers. This is not a contradiction.

I accept that. The contradiction comes about if the one notion
suggests equality of size and the other notion suggests inequality. Which
they do, so there is a prima facie paradox.

The problem is that you are using a word 'size' that you have
not defined.

True. I took it that people knew what I meant. And I think they do.

I sense a cavalierness about common sense intuitions amongst
mathematicians (I don't mean you in particular, Stephen, it's just a
general comment.) Yes there is such a thing as conventional, accepted,
unexamined wisdom. Things are not always what they seem. But common
sense
is, quite literally, where we all start. The articulation of it is
something else.

"Common sense is the collection of prejudices acquired by age eighteen."
−− Albert Einstein.

Common sense is often wrong. Just think where physics would be
if people relied on common sense.

Already conceded.

The problem here is not so much common sense, as the use of
the word 'size' without first defining what you mean by 'size'.

Either a proper subset of a set can be the same size as the set
(for comparable sets or whatever technical qualification is needed), or it
must be smaller than the set, or it makes no sense to compare infinite sets
for size. (I suppose there could be some weirder alternative, such as the
size of a set might depend on how it is ordered, or something like that.
Haven't thought much about that.) Which is it, and why?

Why use the word size at all? Two sets have the same cardinality
if there exists a one−to−one correspondence between them. A set x

> is a proper subset of a set y if every element of x is an element of y,
> and there exist elements in y that are not in x. Those are two
> simple definitions that apply to any two sets.

I know that already.

> Of course people often use 'size' informally to mean 'cardinality'.
> In the finite case 'cardinality' corresponds exactly with the
> common sense notion of 'same number of elements'. Of course 'size'
> need not mean 'same number of elements' even in the finite case.
> Size is a very vague word, even when talking about physical objects.
> Does it mean height, weight, volume? If you use vague words, you
> are going to get vague results.
>
> > First option because Cantor says so might in a way be true, it
> > might be that that is where mathematicians are, but it I was going to join
> > them I would want to know why.
> >
> > Thanks, Six Letters
>
> Noone is doing anything because 'Cantor says so'. Childish comments
> like that are a sure way to make this thread degenerate.

Certainly I write things in the heat of the moment which I later regret. But this wasn't meant as a cheap jibe. I've already conceded that following Cantor might in some deep way be right, if it comes down to following productive branches and forsaking dead ends.

Look at what you've written. It consists of repeating things I already know (definitions etc.) coupled with the suggestion that I'm mixing up different notions of size. Saying that people are confusing two different notions of X is a classic manoeuvre of 20th century philosophy in the moribund analytic movement, and in every case, I'd venture to say, it sells the argument short. As if anybody that disagreed with your point of view was a complete idiot.

There is an intuition that there are less squares (even numbers, primes, whatever) than naturals. We are talking here precisely of intuitions about infinite sets. It is not good enough to say: You're getting mixed up with finite sets, or: You can't rely on common sense intuitions in maths.

So if there are less squares than naturals, then since they have the same cardinality, how can cardinality have anything to do with size (how many)? Why not just say there's a bijection and forget about cardinality.

You suggested I conduct my argument without using the term 'infinity'. I am quite happy to do that. I suggest you conduct the rest of your argument without using the term 'cardinality'.

Thanks, Six Letters
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