Re: Cantor Confusion

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On 12/5/2006 9:20 PM, Virgil wrote:

> > > Except that countable and uncountable coexist within the same set theory
> > > and rational and irrational coexist within the same real umber field.
> >
> > Cantor's DA2 illustrates that there is no such field/list of real numbers.
>
> EB conflates "list" with "set". Nothing in any axiomatic set theory I am aware of requires sets to be lists, or even listable.

Lists are countable sets.

> > Isn't this "coexistence" on the same low level of abstraction a basic though hard to unveil intentional mistake by Dedekind?
>
> What "coexistence"?

I refer to the necessary distinction between something concrete and the abstract name of it. Real numbers are not really numbers in so far, they do not have an accessable numerical address. This is obvious for irrational numbers. Embedded rational ones have to embrace the same properties. They cannot maintain countability. They have to conflate within the entity of continuum. I say, Weyl was wrong in that respect.

> > Dedekind argued: As naturals can be extended to the integers in order to allow subtraction and include negative numbers, and integers can be extended to rationals in order to allow division and include fractions, so rationals can perhaps be extended to reals in order to allow non−linear operations and include irrationals.

> As Dedekind (and others) demonstrated precisely how to construct each of these extensions, his arguments conclude with Q.E.F.

Did you mean Euclid's sentence quod errat demonstrandum (q.e.d.)?
Dedekind wrote in "Was sind..." w.z.b.w. (Was zu beweisen war)

In "Stetigkeit...." we find "was zu beweisen war" only in §5 und §7.

§1 claims: If "a" is an addressable (Dedekind wrote "bestimmte") number, than one can attribute all further numbers of the system R either to a class A_1 of smaller numbers or to a class A_2 of larger numbers.
As long as "a" is addressable, this is correct.

§2 relates to analogy between rational numbers and points of a line.
While Dedekind incorrectly wrote "Analogie zwischen den rationalen Zahlen und den Punkten einer Geraden", he nonetheless understood that the rational numbers do not yet entirely represent the line.

§3 "Stetigkeit der geraden Linie", claims: The Line L is indefinitely richer in terms of (reicher an) point−individuals than R, the set (Dedekind wrote "Gebiet") of rational numbers, in terms of number−individuals. Notice: Dedekind carefully avoided using relations like more or larger. He wrote richer. Probably he felt that a quantitative comparison is not justified.
Having claimed that one can divide the line into points left and right to the cutting point a, Dedekind jolly admits that he does not have a proof that justifies this assumption. However, he conceals that a line can not at all be resolved into a finite number of points.
Well, we imagine: The line can nonetheless be cut into left and right.
So we have to carefully look what Dedekind derives from the banality that there are points left and right. The question is: To what extent are the points addressable.

§4 deals with an irrational square root.
There is very old evidence for geometric ratios to be incommensurable i.e. not expressable by means of a ratio p/q of finite integers p and q.
You all know, sqrt(2)=p/q can be shown to be irrational because 2q^2=p^2=4z^2 is even. Both sides can be divided by 2 but must not have a common divisor.−−> Contradiction.
Dedekind assumed D to be positive and an integer but not the square of an integer. He argues N^2<D<(N+1)^2 with N=pos.integer. etc.
In other words, he considered any single irrational addressable via the

problem that defines it but cannot be resolved numerically. If so, then it is not the cut (A_1, A_2) but the problem alias non−linear relationship which creates irreal "numbers". Dedekind did not create a single new irreal number while any non−linear function "creates" an uncountable continuum of them.

> > Being mislead by the idea of a dotted line of numbers, he overlooked two aspects. First of all, the irrationals cannot be located numerically.

> The "cuts" can be defined precisely.

.... just like a declaration of an intention with no avail and therefore no possibility to be checked for correctness.

> Locating the numbers exactly on a number line may be a a problem for engineers, but is not one for mathematicians.

While it is not a problem to engineers, only dull mathematicians ignore that real numbers must _not_ have an approachable numerical address. Otherwise they were rational numbers.

> > Secondly, the irrationals are not an addendum to the reals but the other way round, the reals vanish completely within the sauce of irreals.

> That "sauce of irreals" is unknown to mathematics.

Concerning sauce, read Hermann Weyl!

The key question is whether or not the embedded rational numbers remain addressable within the genuine continuum. Do not ask pure mathematicians for that. Perhaps they are not in position to understand subtleties. While ane can embed the natural numbers into the integers, positive integers have a sign. Naturals have not.
Likewise, a complex number has to be composed of real part and imaginary part. A complex number with empty imaginary part must still require as many memory as with non−zero imaginary part. Otherwise it is just a real number.
Likewise, a real number has to have no addressable numerical representation because only this makes it different from a rational number.

> What engineers cook up is not our problem.

You are not a mathematician but as narrow−minded as is set theory.
H. Heine, ein Wintermaerchen: Das halbe Fuerstentum Bueckeburg blieb mir an Fuessen kleben (Half principality of B. stuck to my shoes, due to wet ground in November).

> > The irrationals are at best fictitious numbers because they do not have an exact numerical representation available.

> All numbers are equally fictitious, mere creations of the mind.

Please, not again this stupidity.

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