Re: Cantor Confusion

Source: http://sci.tech−archive.net/Archive/sci.math/2006−12/msg03044.html

- *From*: Tony Orlow <tony@xxxxxxxxxxxxx>
- *Date*: Sat, 09 Dec 2006 16:04:00 −0500

David Marcus wrote:

> Han.deBruijn@xxxxxxxxxxxxxx wrote:
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>> stephen@xxxxxxxxxx schreef:
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>>> Han de Bruijn <Han.deBruijn@xxxxxxxxxxxxxx> wrote:
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>>>> stephen@xxxxxxxxxx wrote:
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>>>>> Han de Bruijn <Han.deBruijn@xxxxxxxxxxxxxx> wrote:
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>>>>>> stephen@xxxxxxxxxx wrote:
>>>>>>
>>>>>>> But everything can be modelled as a set.
>>>>>>
>>>>>> Define "everything" and prove that claim.
>>>>>
>>>>> By "everything", I meant everything mathematical. Of course that is not 100% precise.
>>>>> And no, I cannot prove it.
>>>>> But so far all the various

objects of mathematics can
be
modelled using set theory.
That is what is meant by set
theory being a foundation
for mathematics. If someone
were to invent something
"mathematical" (whatever
that may
mean exactly) that could not
be described in terms of set
theory, then set theory
would
no longer serve as a
foundation. But given that
the basics such as the real
numbers,
functions, limits, calculus,
etc. all can be founded in set
theory, it would have to
be something strange
indeed. Not that there is
anything wrong with
strange, but you
probably would like it less
than set theory.

Correction. By "everything" you probably
mean "everything according to
nowadays mainstream mathematics", which
_is_, of course, "mathematics",
according to your probably rather limited
view. But since you can not
really prove anything of the kind, I will rest
my case.

It's not much of a case. You have not presented any evidence
that there exists
any sort of mathematics not describable by set theory. Until
such evidence
exists, the hypothesis that mathematics can be modelled with
set theory has
not been falsified. And don't bother presenting something
that uses limits,
functions, etc. as all of those things can be modelled with set
theory.

Ah, now you are trying to put the burden on me. But that is false play, of course. You said something like "all heat is phlogiston". I do not have to argue that this is not so. The burden remains yours. I didn't even say that set theory is useless within mathematics. I've only said

> > that there's more to mathematics than set theory.
>
> All the mathematics I've ever seen in a math class or read in a math book or journal or done myself can be done in ZFC. Admittedly, there are large areas of mathematics that I know little or nothing about. Still, that would seem to be quite a bit of evidence right there for the statement that ZFC can be used as a foundation for mathematics. So, the burden is now on you to show some mathematics that can't be done in ZFC.
> <snip>

Hi David −

Is it sufficient to show that there are conclusions derived from application of set theory that may not be mathematically correct in all senses? If a conclusion based on premises of set theory does not match the conclusion based on other mathematical methods, then is there not a contradiction between the premises, and therefore premises which are not subsumed under set theory?

The infinite staircase comes to mind, where point set topology considers the limit of the staircase from (0,0) to (1,1), as the number of steps increases without bound, to be the same object as the diagonal line from (0,0) to (1,1), since the locations of the corresponding points become arbitrarily close. This produces a contradiction in measure, the object being of length 2 for all staircases, but of length sqrt(2) for the diagonal line. While the locations of the points in each set approach each other with no lower limit, the directions of the corresponding sub−segments of the two objects are always at a 45 degree angle to each other, producing the error of sqrt(2)/2, the cosine of that angle. So, what we have are a diagonal line of length sqrt(2) and a fractal "line" or curve of length 2. In other words, characterizing the objects as sets of points misses the distinction between the objects in terms of measure, whereas characterizing them as sequences of segments preserves the distinction in terms of direction and overall length.

Now, sequences may be said to derive from ordered sets, but sets are said to be determined solely by membership, with order unimportant. So, the notion of a sequence derives really from an inductive definition such as Peano's, and not from the one primitive in set theory, membership, alone. The notion of order is not captured by "is an element of". Do you disagree?

Tony
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