Re: Cantor Confusion

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- *From*: Tony Orlow <tony@xxxxxxxxxxxxx>
- *Date*: Sat, 09 Dec 2006 19:52:22 −0500

David Marcus wrote:

> Tony Orlow wrote:
>
>> Is it sufficient to show that there are conclusions derived from application of set theory that may not be mathematically correct in all senses?
>
> No. You have to present mathematics that can't be formalized in ZFC.
>
>> If a conclusion based on premises of set theory does not match the conclusion based on other mathematical methods, then is there not a contradiction between the premises, and therefore premises which are not subsumed under set theory?
>
> No. You have to present mathematics that can't be formalized in ZFC. Simply present your "other mathematical methods". Then we can see if they really need techniques that can't be modelled in ZFC.
>
>> The infinite staircase comes to mind, where point set topology considers the limit of the staircase from (0,0) to (1,1), as the number of steps increases without bound, to be the same object as the diagonal line from (0,0) to (1,1), since the locations of the corresponding points become arbitrarily close. This produces a contradiction in measure, the object being of length 2 for all staircases, but of length sqrt(2) for the diagonal line. While the locations of the points in each set approach each other with no lower limit, the directions of the corresponding sub−segments of the two objects are always at a 45 degree angle to each other, producing the error of sqrt(2)/2, the cosine of that angle. So, what we have are a diagonal line of length sqrt(2) and a fractal "line" or curve of length 2. In other words, characterizing the objects as sets of points misses the distinction between the objects in terms of measure, whereas characterizing them as sequences of segments preserves the distinction in terms of direction and overall length.
>
> Present your mathematics by itself. Then we can see if you are using something other than

what is in ZFC.

> Now, sequences may be said to derive from ordered sets, but sets are said to be determined solely by membership, with order unimportant. So, the notion of a sequence derives really from an inductive definition such as Peano's, and not from the one primitive in set theory, membership, alone. The notion of order is not captured by "is an element of". Do you disagree?

Of course I don't agree. You seem to be saying that infinite sequences can't be handled in ZFC. Since ZFC has no trouble modeling the natural numbers and defining functions, it clearly has no trouble acting as a foundation for all of calculus and analysis.

Is there not a single primitive in set theory, namely, e (element of)? How is order derived from that, and why is it not applicable in the case of the staircase?

For any given n, the number of steps, the staircase is defined as the sequence of segment offset pairs:
(x=1−>n: {(0,1/n),(1/n,0)}

The diagonal may be divided into corresponding pairs of segments with the same overall segment offset:
(x=1−>n: {(sqrt(2)/2n,sqrt(2)/2n),(sqrt(2)/2n,sqrt(2)/2n)}

The segments in the first are always vertical or horizontal, while those in the second are all diagonal. The point set interpretation does not catch this. Why?
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