

# Re: Cantor Confusion

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  - *Date:* 9 Dec 2006 21:04:30 -0800
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David Marcus wrote:

Tony Orlow wrote:

<snip>

The infinite staircase comes to mind, where point set topology considers the limit of the staircase from (0,0) to (1,1), as the number of steps increases without bound, to be the same object as the diagonal line from (0,0) to (1,1), since the locations of the corresponding points become arbitrarily close. This produces a contradiction in measure, the object being of length 2 for all staircases, but of length  $\sqrt{2}$  for the diagonal line. While the locations of the points in each set approach each other with no lower limit, the directions of the corresponding sub-segments of the two objects are always at a 45 degree angle to each other, producing the error of  $\sqrt{2}/2$ , the cosine of that angle. So, what we have are a diagonal line of length  $\sqrt{2}$  and a fractal "line" or curve of length 2. In other words, characterizing the objects as sets of points misses the distinction between the objects in terms of measure, whereas characterizing them as sequences of segments preserves the distinction in terms of direction and overall length.

Present your mathematics by itself. Then we can see if you are using something other than what is in ZFC.

Now, sequences may be said to derive from ordered sets, but sets are said to be determined solely by membership, with order unimportant. So, the notion of a sequence derives really from an inductive definition such as Peano's, and not from the one primitive in set theory, membership, alone. The notion of order is not captured by "is an element of". Do you disagree?

Of course I don't agree. You seem to be saying that infinite sequences can't be handled in ZFC. Since ZFC has no trouble modeling the natural

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numbers and defining functions, it clearly has no trouble acting as a foundation for all of calculus and analysis.

But ZFC does have considerable difficulty dealing with infinite sequences, when the Orlovian axioms are added in. I think that Tony's "infinite induction" thing means for a start that if a sequence of elements has a particular property ("staircase length is 2") then the limit of the sequence must have the same property. There doesn't seem to be a definition of the Orlovian limit, except that in any particular case Tony will construct an ad hoc story to make something having the property being talked about. Thus the Tlimit of the staircase sequence is a [search the archive for TO's words] "sort of infinitesimal staircase-thingy of length 2".

I don't think ZFC will handle Orlovian "positive infinite quantities" too well, either. Tony gets to infinite values by simply advancing along the real line for, um, an infinite distance, through the tunnel of love (where it's too dark to see properly). Despite the fact that any finite quantity (integer) can be represented as a "finite length" two-ended string of digits, and the fact that each new integer is formed by adding one, the naive expectation that this would enable a proof by induction that these "infinite quantities" were also simply finite quantities, it doesn't work like this, because, um, there's a principle of somethingorother that excludes this. I wonder if in fact just as electromagnetic radiation is mediated by photons, induction is mediated by inductons, and the tunnel of love just happens to block the passage of inductons. Perhaps.

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