

Re: Cantor Confusion

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- *From:* Tony Orlow <tony@xxxxxxxxxxxxxx>
 - *Date:* Sun, 10 Dec 2006 08:57:32 -0500
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David Marcus wrote:

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Now,
sequences
may be said
to derive
from
ordered
sets, but
sets are said
to be
determined
solely by
membership,
with order
unimportant.
So, the
notion of a
sequence
derives
really from
an inductive
definition
such as
Peano's, and
not from the
one
primitive in
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membership,
alone. The
notion of
order is not
captured by
"is an
element of".
Do you
disagree?

Of course I don't agree. You seem to be saying that infinite sequences can't be handled in ZFC. Since ZFC has no trouble modeling the natural numbers and defining functions, it clearly has no trouble acting as a foundation for all of calculus and analysis.

Is there not a single primitive in set theory, namely, \in (element of)?

Sure. But that just says that there is only one relation that is built into the language of ZFC. We are perfectly free to define new stuff, just as we do in any math class or book.

How is order derived from that,

In the usual way. If we model the natural numbers as $0 = \{ \}$, $1 = \{0\}$, $2 = \{0,1\}$, then we can define $n < m$ to mean n in m . We then define \mathbb{Z} , \mathbb{Q} , \mathbb{R} in the usual way from \mathbb{N} and define addition, multiplication, and order for all of them. Haven't you seen the constructions?

Ugh, yes, I guess I have. The von Neumann ordinals appear to be the vehicle connecting set membership and order this way. Okay. I don't like it, but it works in its way.

It is just supposed to work. No one is saying zero really is the empty set (whatever "really is" means).

Then no one is saying the von Neumann successor ordinals "really are" the naturals? Good.

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It seems like it would be better to have another primitive, such as Peano's successor, than to use this strange definition of the naturals, but I'll have to think about that.

The idea is to be parsimonious. Since you can define the relation you want, there is no reason to be redundant by including another primitive. Doing so just makes things more complicated without any benefit.

Not when the alternative is a construction that establishes the equivalent of a new primitive through a dubious connection between succession and containment. What is more "parsimonious" in inventing some weird model of the naturals and declaring that it exists, rather than having `succ()` be a primitive relation? I don't see the advantage.

and why is it not applicable in the case of the staircase?

For any given n , the number of steps, the staircase is defined as the sequence of segment offset pairs:

$$(x=1 \rightarrow n: \{(0, 1/n), (1/n, 0)\})$$

What do you mean "segment offset"? If $n = 2$, then you wrote something like

$$(0, 1), (1, 0), (0, 0.5), (0.5, 0)$$

You are reading it as if it said $(x=1 \rightarrow n: \{(0, 1/x), (1/x, 0)\})$, but $1/n$ is constant for each iterated value of x . It would be $(0, 0.5), (0.5, 0), (0, 0.5), (0.5, 0)$. That is, up $1/2$, right $1/2$, up $1/2$, right $1/2$.

Oh. Simpler to just write the coordinates of the points as functions of x .

No, that is no simpler, and does not capture the direction or magnitude of any segment in a single pair. That's the "point". The pairs can denote absolute x and y coordinates for a point set perspective, or changes in x and y coordinates that describe the segment in a pair, the starting point of each segment determined by all previous segments, for a segment sequence perspective. Understand?

In other words, the pair of numbers denote the x and y offsets from the start of the segment to its end.

Do you mean the staircase is the path connecting the points

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$(0,0),(0,0.5),(0.5,0.5),(1,0.5),(1,1)$

?

Yes, that's what I meant, and that's what I said. We start at the origin, $(0,0)$. Then we add $1/n$ to the y value to get the next point, $1/n$ to the x value to get the next, and repeat that n times, until we get to $(1,1)$. See? Each segment in the curve is defined by a single pair, the starting point being already defined as the last ending point, and the overall starting point being the origin.

The diagonal may be divided into corresponding pairs of segments with the same overall segment offset:

$(x=1 \rightarrow n$:

$\{(\sqrt{2}/2n, \sqrt{2}/2n), (\sqrt{2}/2n, \sqrt{2}/2n)\}$

The segments in the first are always vertical or horizontal, while those in the second are all diagonal. The point set interpretation does not catch this. Why?

What "point set interpretation"? What doesn't it catch?

I refer to the point set interpretation of the limit of the staircase, put forth some time back by Chas as a counterexample to infinite-case inductive proof. By his argument, since the points in the staircase become arbitrarily close to the corresponding points in the diagonal as n grows without bound, the staircase "in the limit" can be considered to be the same object as the diagonal line. His conclusion is that, since one can prove inductively that the length of the staircase is 2 for every value of n , that proof only applies in the finite case, since his version of the infinite case obviously has a length of $\sqrt{2}$. My response to that was that the staircase in the limit is clearly a different object than the diagonal line, preserving its right angles on the infinitesimal scale, and therefore not straight, but a kind of fractal object. The fact that the segments of the diagonal in the limit are always at a 45 degree angle to the diagonal accounts for the discrepancy in measure of $\sqrt{2}$, the inverse of the cosine of the angle. In other words, approximating the diagonal using the staircase segments cannot provide accurate measure, because the segments are not parallel to the object they are approximating.

This doesn't have anything to do with whether set theory can be used as a foundation for mathematics. You need to give a precise definition of what sort of limit you are doing that would allow the staircase to approach your "kind of fractal object". If you want such a thing, you have to invent it. Developing new mathematics isn't easy.

If the sequence consists of segments of the form $(0,x)$ or $(x,0)$, there is no segment which is diagonal in direction. If every segment in the sequence is of the form (x,x) , there is no segment which is not. This

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information is not evident when the pairs describing the curve are locations, because locations don't have direction.

So, my question remains. If this is a valid formulation of the two objects, and an explanation for Chas' counterexample to infinite–case induction, where does this fit with set theory? I don't think the von Neumann ordinals as a model of a sequence can suffice, since they only allow finite values until a leap is made to the limit ordinals, and continuity is violated. This is the problem I have with the vNO's, and a large part of my problem with transfiniteology. The notion that a sequence must be "countable" simply is not correct in the bigger picture.

So, the point set approach has that the same object has two different measures, because it cannot distinguish between two objects which are locationally the same, and directionally different.

Then come up with an approach that does what you want. By "approach", I mean definitions (of objects, convergence, etc.) that let you prove the theorems you want. That's what everyone does. For example, Einstein came up with Brownian motion, but it wasn't clear how to mathematically model it. Weiner figured out how.

Well, I'm suggesting a definition of the curve as a sequence of pairs which denote xy offsets, rather than a set of pairs of xy coordinates. Is that not a concrete enough description of an "approach" to spark a new neuron in your head? It should be. If you think there is something wrong with it, please elucidate.

I'm sure that cleared things up for you, eh?

Pretty much.

Did it? I rather thought you'd accuse me of being totally nonsensical, though I know I'm not. Nice surprise (unless you're being as sarcastic as I was).

Tony

PS – No good counterexamples to infinite–case induction? Too bad. :(

By the way, the only other counterexample to infinite–case induction suggested made obvious use of a discontinuity based on a buried difference with a limit of 0 in the infinite case, and thus violated the rules as I put forth. So, if you happen to have a good counterexample to infinite–case induction that doesn't rely on such differences, I'd be happy to decompose them for you. :)

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