Re: Cantor Confusion

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- *From*: Tony Orlow <tony@xxxxxxxxxxxxx>
- *Date*: Tue, 12 Dec 2006 10:43:43 −0500

Dik T. Winter wrote:

In article <457d8cc0$1@xxxxxxxxxxxxxxxxxxxx> Tony Orlow <tony@xxxxxxxxxxxxx>
writes:
> Dik T. Winter wrote:
> > In article <1165761763.908889.34550@xxxxxxxxxxxxxxxxxxxxxxxxxxxx>
Han.deBruijn@xxxxxxxxxxxxxx writes:
> > ...
> > > Let P(a) be the probability that an arbitrary natural is divisible by
> > > a fixed natural a. Then P(a) = 1/a . Forbidden by set theory.
> > > > No. Not specifically forbidden by set theory. Forbidden because there are
> > no appropriate definitions for the words you are using (they are not used
> > conforming to standard definitions, so you better supply definitions).
> > In probability theory (as is commonly use) you have to define how you
> > *select* your arbitrary natural. You have not done so, so probability
> > theory does not have an answer.
> > Why does that matter?

It does matter because if you do not properly define your problem,
mathematics is not able to give an answer.

It's sufficiently defined if one assumes that there is a uniform probability distribution.

> This is the same thing as your stupid ball and > vase trick. Why do you need to label anything, or know what you're > choosing from the infinite set?

Because that is part of the problem setting. Giving that setten will
allow mathematics to model the question and give an answer.

That problem has a clear answer with or without the labels: the sum diverges as f(n)=9n. The labels are confounding, not clarifying.

And it is bad to think that because for a sequence of sets holds that

> lim{n -> oo} |S_n| = k
> with some particular value of k, that also
> | lim{n -> oo} S_n | = k
> because the latter statement contains something that has not been
> defined in mathematics.

I'm not sure what that statement is supposed to say. Can uoi give an example?

But even when we define it, it is not certain

> that it holds. Given the following (I think reasonable) definition:
> lim{n -> oo} S_n = S

So, what, S_n is supposed to be an initial segment of the sequence?

> if:
> (1) for every element a in S there is an n0 such that a is in each of
> the sets S_n with n > n0
> (2) for every element a not in S there is an n0 such that a is not in
> each of the sets S_n with n > n0.

In (2), it sounds like a would not exist in ANY S_n if it's not in S.

> So from some particular point an element either remains in the sets in
> the sequence or remains out of the sets.

You mean, at some point you can tell whether a given element a is in S, because if it were, it would be there by then?

> With this definition (when we look at the rationals) we have that
> lim{n -> oo} [0, 1/n] = [0]

Okay that interval degenerates to 0....

> and so:
> lim{n -> oo} | [0, 1/n] | = aleph0 != 1 = | lim{n -> oo} [0, 1/n] |
> (I am talking standard mathematics here).

Are the |'s supposed to denote set size? If so, how can you claim that [0,0] contains aleph_0 elements?

> So taking cardinality and limits can not be interchanged except in some
> particular cases. But that is not unprecedented in mathematics.

> limits and integrals can also not be interchanged except in particular
> cases. And so can the interchange is not in general passoble if one
> of the things you interchange is a limit. Even interchanging limits
> is not in general possible. Consider:
> lim{x −> oo} lim{y −> oo} (2x + 3y)/xy

True, but is it relevant?
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