Re: Placing Balls in Urns and Expected values

Source: http://sci.tech−archive.net/Archive/sci.math/2006−12/msg05260.html

---

- *From*: "kunzmilan" <kunzmilan@xxxxxxxx>
- *Date*: 17 Dec 2006 08:28:04 −0800

C6L1V@xxxxxxx napsal:

> Rob Johnson wrote:
>
> > In article
> > <1166352415.987805.284770@xxxxxxxxxxxxxxxxxxxxxxxxxxxx>,
> > "kunzmilan" <kunzmilan@xxxxxxxx> wrote:
> >
> > > There are n^n possibilities how to place n indexed balls into
> > > n indexed
> > > urns, if balls are indexed, too.
> > > From this number, there are n possibilities how to place all
> > > balls into
> > > only one urn.
> >
> > This agrees with [1] from my previous post; since k = n−1, only the
> > term with j = n−1 is non−zero:
> >
> > −−− j−k n
> > > (−1) C(j,n−1) C(n,j) (n−j)
> > −−−
> > j
> >
> > 0 n
> > = (−1) C(n−1,n−1) C(n,n−1) 1
> >
> > = n
> >
> > > We have n!/(n−2)!1!1! how to chose two urn to place odd
> > > number of balls
> > > in them, and eventually n!/(n−2)!2! , if the number of balls in
> > > both
> > > urns is the same. These counts do not count with indexing of
> > > balls and
> > > leave (n−2) urns empty.
> > > If only urns are indexed, the sum of all such coefficients for

> > > all
> > > partitions reduces to
> > > (2m −1)!/m!(m−1).
> > > The sum of both coefficients divided by this coefficient gives
> > > the
> > > probability of (n−2) urns to be empty.

> > To compute probability, we must index the balls. Consider the case
> > where n = 2; 2 balls and 2 urns. If we do not index the balls, there
> > are 3 arrangements:
> >
> > |o| | | | | | | | | |o|
> > |o| | | |o| |o| | | |o|
> > − − − − − −
> >
> > but they are not equally likely. Coloring the balls (b for blue, r
> > for red) does not change the probabilities, however, it does show
> > the proper probabilities; each ball can be in each urn with equal
> > probability:
> >
> > |b| | | | | | | | | | | | | |b|
> > |r| | | |b| |r| |r| |b| | | |r|
> > − − − − − − − −
> >
> > That is, each ball will be in the left urn 50% and the right urn
> > 50%, as shown above.
> >
> > So labelling the balls determines the proper probabilities.

> Very nice little demonstration. Of course, that only holds for
> "classical" balls; for indistinguishable quantum−mechanical balls, the
> first (unlabelled) version holds for bosonic balls, and only the middle
> configuration of the unlabelled version is possible at all for
> fermionic balls.
>
> R.G. Vickson

The distinction is not given by properties of balls, bosonic or
fermionic, but on the way, how we count. For example, 3 balls (i index)
and 3 urns (j index):
(1,0,0)
(0,1,0)
(0,1,0)
The column sums (eventually diagonal elements in the quadratic form)
are (1,2,0). Any distinction vanishes.
Ones in the matrix are distinguished only by their position.
It is necessary to use the second polynomial coefficient to take
account on the position of ones in columns (m−permutations).

If ones have their index (are distinguishable), the third index must be used
(1_a,0,0)
(0,1_b,0)
(0,1_c,0).
Instead of n^m possibilities, the solution of this problem leads to the growing factorial function.
kunzmilan

.