Re: Small Set theory:Revised.

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- *From*: "zuhair" <zaljohar@xxxxxxxxx>
- *Date*: 20 Dec 2006 19:26:21 −0800

hagman wrote:

zuhair schrieb:

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hagman wrote:

Now
I
have
a
question,
should
I
add
an
axiom
stating
that
for
every
set
there
is
a
P
that
defines
it.

How would
you proof
~xex
without it?
If one wants
to use Ax.3
to show that
~xex, one
needs that x
is
P_embedded
(shudder).

Why, all what we need to
apply 3. is x is P_defined.

Exactly. I meant to say that *without* an
additional axiom that every
set is P_defined for some P, you cannot
simply apply ax.3 to prove ~xex
holds for all x.
But *if* you add that axiom, your first
theorem (universe) becomes
false because ~vev follows.

This doesn't stop it from having a universe, since Ax.3 will
forbid v
from being a member of any set, i.e v here will be the only
proper
class in this set theory, while at the same time we have all
other sets
as members of v. Therefore still the universe theorum holds.

Zuhair

if I put Ax.2 and Ax.3 in one axiom then this will vanish. I mean we
can have
a universe.

There's no difference between

Ax.1
Ax.2
Ax.3
Ax.4

and

Ax.1
(Ax.2 & Ax.3)
Ax. 4

ah I see let me see. let me restate the axiomatic system again.

−Small Set Theory−

Primitive e

Definition:
x is P_defined <−> Ay (((P[y]−>yex)&(~P[y]−>(~yex&~y=x))).
x is P_embedded <−> Ay(P[y]<−>(yex v y=x)).
x is P_not embedded <−> Ay (P[y]<−> yex).

were P is a formula in one free variable.

Also we can define x is P_defined in the following manner

x is P_defined <−> (x is P_embedded v x is P_not embedded)

i.e.

x is P_defined <−> Ay((P[y]<−>(yex v y=x))v(P[y]<−> yex)).

Axioms:−

Ax.0) Extensionality: As in ZFC.
Ax.1: AxEP (x is P_defined)
Ax.2: AxAy (yex & y is P_defined −> ~P[x])
Ax.3: AxAy (((P[y]−>yex)&(~P[y]−>(~yex&~y=x)) & y is not P_defined<−> x Exist).
Ax.4: Infinity: as in ZFC.

+/− AC.

Now your argument was that Ax.1 will refute the existance of a univeral set.

Let V={x|x=x} , i.e. V is the set of all sets , i.e. the universe in this theory.

Let P[y]<−> y = V.

by 1. you thought we should have x is P_defined should exist. But this is not 1. Axiom actually mean that for every already existing x there should EXIST a formula P that determine membership in x and non membership in x according to the definition of x is P_defined given above. Not for every formula P there should exist x such that x is P_defined as you thought.

There cannot exist x such that V is a member of it. This is a consequence of Ax.2

Because by Ax.2 any set which include V should not be equal to itself. and thereby not a set, since it violates extensionality.Also Ax.3. which is the existance source of sets in this theory do not allow y when y is P_defined to be a member of x if x is P_defined.

Therefore Ax in Axiom 1 do not extend to a set that has V as a member, because simply this set do not exist in this theory.

I think there is no problem with the theorum of Universe, Pairing, Union, specification, Replacement, power. in this theorum (after suitable modefication to suit this axiomatic system of course).

I think this theory would meet the objective I wrote before behind developing this theory. A theory that avoids Russell's paradox with the most minimum acheivable lose of models.

Zuhair

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