Re: On The Fundamental Theorem of Arithmetic and Why it Breaks Down for the Algebraics

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In article <870j64−nsg.ln1@xxxxxxxxxxxxxxxxxxxxxxx>,
The Ghost In The Machine <ewill@xxxxxxxxxxxxxxxxxxxxxxxx> wrote:

> I have a dumb question, and am not sure precisely how to phrase it to Google.
>
> As everyone in this forum should know, for every positive integer N, one can find a unique decomposition into primes such that
>
> N = 2^e_2 * 3^e_3 * 5^e_5 * 7^e_7 * ... * p ^ e_p
>
> where e_2, e_3, e_5, etc. are nonnegative integers, and p are special positive integers usually called primes, divisible by only themselves and 1.
>
> Usually the zero exponents are omitted, so that one get things such as
>
> 33 = 3 * 11
> 42 = 2 * 3 * 7
> 54 = 2 * 3^3
> 65 = 5 * 13
> 109 = 109^1
> etc.
>
> This is of course the Fundamental Theorem of Arithmetic, proven long ago by either Euclid or Gauss. I'll admit to not being familiar with Ernst Kummer's work but am curious as to why this factorization fails entirely in the ring of algebraic integers, which are, of course, those roots (real or complex) for irreducible members of Z[x] whose highest power term has a coefficient of +/− 1.

In the full ring of algebraic integers, there are no "primes" (there are no irreducible elements). So you cannot have unique factorization into primes; in fact, you have NO factorization into irreducibles for

any algebraic integer other than units (which have the empty
factorization).

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"It's not denial. I'm just very selective about
what I accept as reality."
--- Calvin ("Calvin and Hobbes" by Bill Watterson)
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