Re: f and f' square integrable

Source: http://sci.tech−archive.net/Archive/sci.math/2007−01/msg00005.html

- *From*: The World Wide Wade <waderameyxiii@xxxxxxxxxxxxxxxxxxxx>
- *Date*: Sun, 31 Dec 2006 14:32:45 −0800

In article <NDHlh.1114208$084.1066831@attbi_s22>,
Stephen Montgomery−Smith <stephen@xxxxxxxxxxxxxxxxx> wrote:

> Fedor wrote:
>
>> Hi all,
>>
>> suppose that f:R^n −−>R is a smooth function such that f^2 and (f ')^2 are integrable over R^n. Is it true that f(x) tends towards 0 when |x| tends towards infinity ? It is easy for n=1 but is it true for the general case ?
>>
>> Regards,
>> fedor
>
> I think not.
>
> First consider a function like f(x)=log(1/|x|)^a for 0<a<1/2. Check that f and f' are square integrable in R^2.
>
> Next consider sum a_n f(x−(2n,0)) where a_n is square summable.

If f(x) = [log(1/|x|)]^a for x near 0, then f is not smooth at 0. Perhaps you can play around with this to arrive at an example, although the obvious things don't seem to work.

I don't have an example for R^2, but if n > 2 you can do this: Take any g in C^oo with support near 0 and g(0) nonzero. For b > 0, set g_b(x) = g(bx). Then int |g_b|^2 = (int |g|^2)/b^n and int |D(g_b)|^2 = (int |D(g)|^2)/b^(n−2). (All integrals are over R^n and D is any partial derivative of f.) Letting u denote any unit vector you like, define f(x) = sum (j=1,oo) g_(2^j)(x − ju). Then f is C^oo, and f and any Df belong to L^2. But f(ju) = g(0) for all j, so f does not tend to 0 at infinity.

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