Re: Irrational numbers questions

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- *From*: David C. Ullrich <ullrich@xxxxxxxxxxxxxxxx>
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On Wed, 03 Jan 2007 12:57:34 +0100, Han de Bruijn
<Han.deBruijn@xxxxxxxxxxxxxx> wrote:

> The following is from an old poster at 'Irrational Numbers Proof':
>
> http://www.newton.dep.anl.gov/askasci/math99/math99119.htm
>
> Quote:
>
> We write ln(N) = a, which means what power "a" must I raise the number "e" in order to obtain the number "N". The answer is always irrational for integers "N" except for N=1, because ln(1) = 0. The function ln(N) may even be transcendental.
>
> May be true. But I wonder how to prove this.
>
> Let's start somewhere. Let it be known that e is irrational. A rather concise proof is found in:
>
> http://en.wikipedia.org/wiki/Proof_that_e_is_irrational
>
> Can it be proven that, with e irrational, also e^N (with N natural) is irrational?

It's true that e^N is irrational for natural numbers N. That doesn't follow from the fact that e is irrational (for example if x = sqrt(2) then x is irrational although x^2 is rational), but it's a fact, a special case of the fact that e is transcendental.

> Guess so, but why .. And is the sum of an irrational and a rational number also irrational? Guess so, but why ..

The fact that the sum of an irrational and a rational must be irrational is on the other hand completely trivial.

> Assume now that ln(N) is rational = m/n . Then e^m = N^n where the left hand side is irrational and the right hand side is rational. Giving a contradiction. Thus ln(N) is irrational. Right?

Yes.

> Does it follow now that sum(k=1..N)(1/k) − ln(N) is irrational?

Yes, because a rational plus an irrational is irratinoal.

> Note: this is an approximation of the Euler−Mascheroni constant.
>
> Han de Bruijn

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David C. Ullrich
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