Re: Q: About number of primes with n digits?

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- *From*: "Danny" <fasttrack2a@xxxxxxxxxxxxx>
- *Date*: 3 Jan 2007 19:03:07 −0800

Chip Eastham wrote:

> Danny wrote:
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> > > Danny wrote:
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> > > > jankrihau@xxxxxxxxxxx wrote:
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> > > > > Danny wrote:
> > > > >
> > > > > > The first 4 primes are single digits in length. The next 21 primes are 2 digits in length. The next 143 are 3 digits in length. etc..
> > > > > >
> > > > > > 4, 21, 143, 1061, 8363, 68906, 586081, 5096876, 45086079, 404204977, 3663002302, 33489857205,

> > 308457624821,
> > 2858876213963,
> > 26639628671867,
> > 249393770611256,
> > 2344318816620308,
> > 22116397130086627,
> > 209317712988603747,
> > 1986761935284574233,
> > 18906449883457813088,
> > 180340017203297174362
> >
> > Sequence is in OEIS as A006879.
> >
> > Will the ratio between terms converge?
> >
> > If the sequence is divergent then at any point can the next ratio be < the previous ratio?
> >
> > Dan
>
> By the PNT, the nth term is asymptotically
>
> 0.9 * 10^n / (n log 10)
>
> so the ratio converges to 10.
>
> ---
> J K Haugland
> http://home.no.net/zamunda

My reasoning is probably way off but if the above formula you give is only asympototically
correct how can the ratio be an absolute convergence

too 10?

I have been looking up the PNT and see nothing
about any value pertaining to the different (n) lengths
of the primes. Only the different methods used for estimating
pi(x) not digit length (n) counting.
Also nothing about ratio convergence of this particular
count.

Then again, I could have over looked something.

Thanks,

Dan

The prime counting function pi(x) is defined to be the number of (positive) primes less than or equal to x, for any real number x. The Prime Number Theorem says simply that pi(x) ~ x/ln(x), but a more precise statement can be given in terms of the logarithmic integral, esp. if one assumes the Riemann Hypothesis.

If the asymptotic character of this result bothers you, a simple restatement is that pi(x)/(x/ln(x)) tends to 1 as x tends to +infinity.

In any case, isn't it evident that the "primes of length k digits" is exactly:

pi(10^k) − pi(10^(k−1))

for integer k > 1?

regards, chip

Yes, it is evident!

In your first statement, even if assuming the Riemann Hypothesis, it is still an approximate value to the real value count x for any given prime (p).(pi(x)=p).

Granted, it does give the best approximation.

Indeed Riemann did give the best approximation, which is an exact formula for pi(x) using all nontrivial zeroes of the Riemann zeta function (assuming the Riemann hypothesis).

> It is like trying to find a closed form for pi(x) which will never happen.

Various exact "closed forms" are known, but pale in efficiency with recursive techniques.

> A good anology is the use of the gamma function for finding the asymptotic value for (n!)

Gamma(n+1) = n! exactly.

> It is sort of ironic that the primes and the factorials are related because of Wilsons' theroem and both are not computable in a closed form method but need a crutch such as the Riemann Hypothesis for pi(x) and the gamma function for (n!) to give the best approximation.
>
> Dan

My point was that J K Haugland's answer (10) to your question about the limit of the ratios of counts of primes with successive lengths (in digits) is rigorously deducible from the PNT.

regards, chip

Thanks chip, I can see that now.
Thanks to J K Haugland also.

This stuff is interesting where right now I am investigating large composites that have two large prime factors of equal digit length with a ratio ~ 2 between the two factors.
Very easy to factor using ECM. Digit length of composite is e+617.

Dan

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