Permutations, Fractions, Primes (a "game")

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First, take a permutation {b(k)} of the first n positive integers, and
then take the partial sums, for 1<=m<=n,

s(m) = sum{k=1 to m} 1/b(k).

After reducing the n fractions {s(m)} so their numerators and
denominators are coprime,
we can get a "score" equal to the total number of primes in both the
numerators and denominators of the s(m)'s.

For example, if, for n=6, we have the permutation
(3,4,1,6,5,2),

then we have the s(m)'s
1/3, 7/12, 19/12, 7/4, 39/20, 49/20.

So we have the primes: 3,7,19,7, for a score of 4.

If we make a game of this, a better player might play the permutation
(2,6,1,4,5,3),

which gives the s(m)'s
1/2, 2/3, 5/3, 23/12, 127/60, 49/20.

So we have the primes: 2,2,3,5,3,23,127, for a score of 7.

Regarding the maximum possible score for each given n:
Joseph Biberstine, as discussed on the Integer Sequence Fan
email group, calculated the first few terms of the sequence:
(a(1) is the first term.)
0, 3, 4, 4, 6, 8, 8, 10, 11

Here is one of the best 6−integer solutions (score of 8):
2,6,1,3,5,4

The s(m)'s:
1/2,2/3,5/3,2/1,11/5,49/20.

I would ask someone to compute more terms so this maximal−score sequence
can be submitted to the EIS, but, as is the case wih many game−derived sequences, the relatively long explanation of the sequence may make it too esoteric to be interesting to a general audience.

Thanks,
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