Re: History of trigonometry

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- *From*: "Ross A. Finlayson" <raf@xxxxxxxxxxxxxxx>
- *Date*: 5 Jan 2007 06:52:00 −0800

Narasimham wrote:

> Ross A. Finlayson wrote:
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> > Narasimham wrote:
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> > > Ross A. Finlayson wrote:
> > >
> > > > Basically my question here is: what's
> > > > 4−gonometry.
> > >
> > > May be the tetrahedronometry.It should include dihedrals
> > > and trihedral
> > > "solid" angles.Unfortunately, this 3D counterpart did not
> > > develop so
> > > well or used so much like plane trigonometry.It could have
> > > involved
> > > quaternions.
> > >
> > > > Why is trigonometry called trigonometry?
> > >
> > > Very clear from its Sanskrit roots: Tria (three,sounds are
> > > similar in
> > > English,German and French),Kona (gonio is angle),Matra
> > > (size,measure
> > > or unit).These three currently used Indian words are common
> > > parlance.
> > >
> > > Narasimham
> >
> > Hello Narasimhan,
> >
> > Thank you, yes, the word roots maintain their generally understood
> > meaning. I didn't know Sanskrit was that close to Greek, only ever
> > having heard of Sanskrit as being a dead language, i.e., no longer

spoken, with a fixed vocabulary and grammar, like Latin or ancient Greek, although somebody added "windsurfing" to Latin some years ago, I thought it was more insular.

Hi Ross__ Almost all Indian languages are rooted in Sanskrit,used in auspicious/priestly functions,and so it may live on for all time.. Once at tea−time my friend and I were preparing a sort of oral compendium of Greek/Latin/Sanskrit words and were pleasantly surprised at so mamy commonalities. Apart from quite similar common first relatives' names (father,mother,brother) = (pitha,maatha,braatha),the first person "I" is like German "Ich" and "Ishtam" in Sanskrit means "Free−self−will, Liking volition", and also e.g.,"Bandha" of Sanskrit and Bund/Band (meaning tie) of German/English they are all almost same words and sense...But my pitch won't go out of certain bounds like . :)..

I research those terms, or rather, "google" them, besides a reference to Lexell with some description of a generalized polygonometry in the late 1700's blind Euler era, they appear to be more about the spherical "trigonometry", I think that has to do with great circles and so forth, and don't know, I'm wondering more about extension of planar goniometry. The polygonometry is referenced with regards to the precession of orbits and so on, I've read a decent textbook on using quaternions for orbital computations. I can't recall it, not having had the background, but it was pretty good. I read some of your posts and am impressed, I wish I was a better differentialist. I'm average. I have here a copy of a Dover reprint of the "Theoretical Kinematics". There's quite some discussion about trigonometry these days, otherwise as usual it seems there is much about foundations.

I wrote a little program last night to graph them, these evolving coordinate systems, I'll try and get an applet together to illustrate this "n−gonometry."

It might be interesting to consider how totally fascinating geometry can be. While that may be so, I'm sure for some it's remarkably boring. Ha ha ha. Consider for example, the interior angle of a polygon is 2 pi radians, except a triangle's interior angle is 1pi radians. Consider the exterior angle, it's (2 pi − 2 pi / n) * n, where n is the number of sides. So, for example, for a polygon of 100 sides, its exterior angle is 198 pi. The exterior angle or sum of exterior angles of a polygon of n sides is (2n−2) pi radians. The radian is the arc length subtended by the angle, of the unit circle. So, the 180 degree angle is pi, radians, that's the length of the perimeter of the circle swept through from the beginning to end of that angle. Fascinating.

The circle, it's 360 degrees or 400 gradients. Nobody uses gradients anymore, and that's hyperbole.

> > So, is the exterior angle of the circle 2 pi? That would be
> > non−archimedean, as it were. In smooth infinitesimal analysis, the
> > circle has infinitely many sides, it is the infinitely sided regular
> > polygon, yet with no corners, is it zero?
> >
> > I'll try and have some applet to illustrate the drawing of these
> > periodic "n−gonometric" functions and perhaps some analytical results
> > of them shortly. Good day.
> >
> > Ross
>
> Before spending any expensive time on applets, one has to get at the
> Euclidean fundamentals right. The sum of all exterior angles of a
> polygon (no matter triangle, polygon or circle) is 2 pi. When sides of
> a polygon are seen as vectors,it just means that total rotation angle
> around a flat point is 2 pi.
>
> You must read Gauss−Bonnet Theorem for a comprehensive geometrical and
> topological insight.Total rotation angle around a flat or elliptic
> point is 2 pi and increases with negative Gauss curvature,i.e., with
> warping of plane around a point.
>
> Cheers and happy new year,
>
> Narasimham

Hi Narasimhan,

Yes, that seems quite the reasonable thing to do, to learn more about geometry if I think there is a novelty in it. If anything is, geometry is _the_ most explored field of the mathematics over the millenia.

About the interior vis−a−vis exterior angles, the interior angle of the regular polygon, n−gon, with n > 3, is 2pi / n. At that vertex, the interior angle's complement, the exterior angle, is 2pi − 2pi/n. Summing that over the vertices, that is 2(n−1)pi. Is that not the definition of interior and exterior angle?

Well I'm coding up an applet, using the "Java" programming language and system. One of the nice features about Java is that there is a largish standard library in the "Java 5", which back in the day was called "Java 2". I figure to use this "Java2D" to display animations and generate figures, so I've implemented a regular polygon class. A problem I find is that there is not an algebraic irrational numeric representation, and casual research does not bring one forward. So, I consider how to implement the storage of coordinates of the vertices of a regular polygon, which would seem to be always algebraic rational or irrational. The algebraic number is a real root of a polynomial with integer or equivalently rational coefficients. The standard library does not include an algebraic numeric data type. So, to exactly record the coordinate list of a regular polygon, there is required an algebraic data type, and some call those ADT's, but those are abstract data types. I figure that I can implement the algebraic point as a list and then the operations have various number−theoretic algorithms on the integer typed or ratio typed list elements.

I wonder what is the "unit" regular polygon. The unit square generally refers to the square with unit area, while the unit circle refers to the circle with unit radius. To variously circumscribe or inscribe the polygon to the unit circle and say it is a unit polygon if the "apothem" or "center to vertex" length is the unit, I wonder if that feature is usable in characterizing these geometric figures.

Then, I wonder how to organize the coordinates for a regular polygon "at rest". One notion is to have the vertex labelled 1, or A, be at the origin and the first edge along the X axis. Another is to have the center of the polygon at the origin, and then have vertex A on the X or Y axis. Another is any of the orientations that fit into the n^2−gon, or nk−gon where k has no factors not in n.

I appreciate that this is simple, but I wonder quite a bit about these periodic functions that are describable geometrically, and their analytical properties, with the differentially ratioed angles. The ratios aren't constant, they're generally periodic and differential themselves.

I browse some reference pages about the Gauss−Bonnet theorem you mention, and it will be a while before I use it. Gauss, Riemann, Euclid, Lobachevsky, they're various geometers, and I know only little of their work.

Thanks for the information about polygonometry.

If the circle is the infinitely sided regular polygon, are the vertices and edges each point width in alternation about it? Heh. In a way, yes, they are, and in a way, yes, they is.

Ross

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