Re: A help with trigonometric functions, please

Source: http://sci.tech−archive.net/Archive/sci.math/2007−01/msg02723.html

- *From*: Denis Feldmann <denis.feldmann.asupprimer@xxxxxxxxxxxxxxxx>
- *Date*: Sun, 14 Jan 2007 19:21:34 +0100

David C. Ullrich wrote

> On 14 Jan 2007 07:17:59 −0800, nicegirl_130@xxxxxxxxx wrote:
>
>> Hi, I'm a 15 yo girl and I like to study math more than required by my school. Some days ago I asked a help with complex numbers, the answers were quite interesting, and then I read about the polar representation of a complex number. I thought I had understood, but then a guy who knows a lot of math told me that, actually, when you write z = r(cos (t) + i sin(t)) and you think about t as an angle, then the representation is not precise, because t is not an angle. I had made a comparisson between z and a vector on the plane, and it was clear to me that the real part r cos(t) was similar of the projection of a vector on the vertical axis and the imaginary part r sin(t) was like it's projection on the vertical axis. That was my geometric reasoning, thinking of t as an angle and of r as a magnitude, a lenght.
>>
>> But the guy told me this is not precise and, in formal Math, is wrong. Though I didn't understand what he meant, I could see this was about the very definition of the sine and cosine functions The definitions I have of these functions are those based on right triangles or, maybe more precise, on the unit circle. But he told me those definitions are not very accurate, because their right definitions of sin and cosin are based on something he called power series. The guy didnt want to waste his time with someone my age and didnt go into details, but I think a power series is something like a polynomial of infinite degree with infinitely many coefficients. I have studied geometric progressions and I know that if |x| <1 then 1 + x + x^2....+ x^n...... = 1/(1−x), that is, when n goes to infinity the sum of the terms of the progression gets as close to 1/(1−x) as desired, though never reaches 1/(1−x). If I guessed right, this is a power series.
>>
>> Well, does this mean everything I have studied so far about trigonometric functions is wrong? Kinda frustrating!
>> Sharon
>
> No, there's nothing wrong with anything you know, as far

as I can see. There's also an important point sort of lying behind what the guy was saying, but if he says no you're wrong, t is _not_ an angle, that's just being silly.

When you learn trigonometry you learn that an _angle_ has a sine. From that point of view you need to specify the units the angle is measured in − I don't know whether you've ever heard of radians, but a radian is a unit of angular measure: pi radians is the same as 180 degrees.

_So_ if you're thinking about the sine of angles then something like "sin(180)" doesn't make sense − you can't talk about the sine of a number, it's the sine of an angle (sin(180 degrees) = sin(pi radians), but sin(pi radians) would be different.)

Now later, especially when you start studying calculus, there are reasons you need to have a definition of sin(x), where x is a _number_, not an angle. So the sine function gets redefined. Let's say the new function is Sin and the one you know is sin. Sin(x) is defined for x a number, not an angle, while sin(t) is defined for t an angle, not a number. It turns out that

Sin(x) = sin(x radians).

So it's kind of obvious that anything you can do with Sin you can also do with sin, and vice versa. The difference is not just silliness, there are good reasons why x has to be a number in Sin(x).

But they're really just two ways of looking at the same thing − saying it's _wrong_ to think of t as an angle in sin(t) is just stupid, you're just talking about sin instead of Sin.

Note that this notation Sin is just something I made up for this post − if you say something to someone about sin versus Sin they won't know what you're talking about.

He's also right, although he's being kind of stupid insisting that his point of view is the only correct one, that the trig functions are best defined in terms of power series. This is because actually _defining_ what an angle _is_ is trickier than you think. It turns out that

Sin(x) = x − x^3/3! + x^5/5! − x^7/7! + ...

Cos(x) = 1 − x^2/2! + x^4/4! − x^6/6! + ...,

where n! = 1(2)(3)...(n).

****

David C. Ullrich

It's bad netiquette to repost it, but I wanted to seize this opportunity of saying what is perhaps obvious, but so often forgotten by people like JSH, Hans de Bruin and others, i.e. that when questions are asked in earnest, people like David Ullrich will go quite a length to give clear and deculpabilizing explanations. Could the sarcastic replies some people get from him be a measure of the quality (or otherwise) of their questioning ?

.