Re: Cantor Confusion

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- *From*: David Marcus <DavidMarcus@xxxxxxxxxxxxxx>
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Andy Smith wrote:

> David Marcus writes
>
> > > But, if something is undefined it is usually because
> > > the coordinate system describing the underlying thing
> > > is inappropriate at that point. e.g. y = mx + c is
> > > not a good basis for describing all lines in a plane
> > > (a better set of coordinates are the minimum distance from
> > > the origin and a unit vector).
> >
> > That is not what "undefined" means.
>
> I think that you miss my point, or discount it. A line parrallel to the
> y axis has an infinite gradient; such a line viewed in (m,c) coordinates
> it has an "undefined" value (inf, inf). But the line is perfectly
> sensible at e.g. x = k. So, the same thing, I think (maybe), applies to
> functions. asin(y) is multivalued, difficult to comprehend,

There is a single−valued version of asin. In fact, all functions are
single valued (although in advanced math there are sometimes
"multivalued functions", but this is an abuse of language).

> but a shift
> of perspective gives y = sin(x), which is readily comprehensible. Ditto,
> possibly, for situations involving infinities large or small − a change
> in variables potentially makes things more comprehensible.

You are saying that different ways of analyzing a situation may have
different attributes. And, some ways may work better than others. Very
true.

> > We say what function we are talking about. Let
> >
> > f(x) = sin(1/x), if x <> 0.
> >
> > g(x) = sin(1/x), if x <> 0,
> > 0, if x = 0.
> >
> > h(x) = sin(1/x), if x <> 0,
> > 1, if x = 0.
> >
> > f is not defined at zero. g and h are defined at zero. Using Wikipedia's definition of "odd", f and h are not odd, g is odd. It is meaningful to ask whether f can be extended to all of R so that its extension is odd. The answer to this question is yes.

> I don't know about Wikipedia − I used the term "antisymmetric". f,g,h are all antisymmetric.

The question is whether your "antisymmetric" requires the domain to be all of R.

How can h be antisymmetric? Doesn't the value at zero break the anti−symmetry?

> g is "obviously" correct − because if you invert
> the universe left to right, there is no reason to suppose that sin(1/x)
> changes sign at x =0. Possibly not a killer argument.

What does "correct" mean?

> > > exp(−1/(x^2)) is also "not defined" at x = 0, but
> > > it would be a mistake to define the function and all
> > > its derivatives as e.g. 42.0 at x=0, and then say
> > > that the function and all its derivatives can be made less
> > > than any value d by choosing some 0<x<eta.

> > It is not a "mistake" to define a function as follows.
> >
> > k(x) = e^(−1/x^2), x <> 0,
> > 42, x = 0.
> >
> > It is true that k is not continuous, but it is a perfectly valid function. Do you know the definition of "function"?

> Doubtless you lot have some legally watertight definition. As far as I am concerned, a function is a formula that provides a value for a given input (or inputs, in a multidimensional situation).

Ah, you are at least a hundred years behind the times. No, a function is most definitely not a formula. A function is a rule which assigns, to each of certain real numbers, some other real number. For example, the rule that assigns to each number a the number 0 if a is irrational and the number 1 if a is rational is a function, but you will have a hard time coming up with a formula (nor is a formula required).

> (incidentally
> how does exp(−1/x^2) get off the ground anyway? OK, it's
> not
> defined at x = 0, but you CAN define a function f by
> e.g. Taylor expansion in x about some x0, such that f(x) =
> exp(1/x^2) for all x!=0, and that IS defined at x =0,
> and that has f and all its derivatives = 0 at x =0 ?)

> What does "get off the ground" mean? I don't understand why you want to use a Taylor expansion to define the function (and I'm not sure if the Taylor expansion you want to use is zero at zero; I think it isn't).

> Well maybe that would explain how it "gets off the ground". Excuse the loose talk, but I thought that would be clear − if a function and all its derivatives are zero at a point, then if you move an infinitessimal distance delta, to order (delta)**2 the function and all its derivatives are still all zero. And in a Taylor expansion to a finite offset, the function is still zero, because all its derivatives are zero.

Yes. The Taylor expansion can't get off the ground. However, the function can.

> Why
> not just do it directly? I.e., let
>
> F(x) = e^(−1/x^2), x <> 0,
> 0, x = 0.
>
> This is a perfectly good function. It is infinitely differentiable, but not analytic. I.e., all of its derivatives exist, but its Taylor series at zero does not equal the function in any neighborhood of zero.

> > Well that is undoubtedly the answer to the question. Why doesn't the
> > Taylor expansion work might be a better way of phrasing the question?

An interesting question. It turns out that the behavior of the function for complex values of x prevents the Tayor series from working for real values.

> > > I think you don't understand how the word "define" is used in
> > > mathematics. What math courses have you had?

> > I am definitely not conversant with your technical definitions of words. Me, as far as Maths courses go, if you want to talk about anything dirty, like applied math, Fourier series, signal processing, vectors and differential equations, I am pretty clued up. But as far as set theory, number theory, pure maths in general, de nada. So from your perspective I am a Klutz. But I am not trying to engage/compete ... just curious, no ego here, happy to admit ignorance. Yes, I should read a book (I am, actually) but that would just result in more questions...

Actually, I take back what I said about the word "define". I think your main problem is that you aren't familiar with the modern concept of function. Functions are no longer formulas. They were in mathematics a couple of hundred years ago, and engineers and scientists still often think of them that way. But, things have changed. The modern concept clarifies things a lot.

You might enjoy looking at the book Calculus by Michael Spivak. I'm sure that it explains Calculus in a way that is very different from how you learned it.

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David Marcus
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