

# Re: Is continuum completely filled up?

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If any 2 real numbers are different, there is a gap between them, in which of course there is another real. But there are also now 2 gaps. So while there are an infinite number of reals there must also be an infinite number of gaps?

Russell wrote:

Here, I think, is the fallacy: you go in one fell swoop from 2 to infinity. What's true for any finite process is not necessarily true for an infinite one.

Plus, I think you have to be crystal clear what you mean by a gap. How would you know if the gaps were still there after you "go to infinity" with the process? You need some kind of procedure for finding a gap. I think if you try to define one, at least if it's one that agrees with our notion of a line with order topology, you'll discover that everywhere you look for a gap, there is instead a real number there.

Indeed, it looks like every time you find a "gap" between two reals, you then find another real point to "fill" it. So if you continue this quest for gaps indefinitely, won't you end up filling them all up and thereby produce a completely "filled" line?

Looking at it another way, for any two unlike reals you can always find a third real (actually an infinite number of reals) that lies between them. So the logical conclusion is that there is no "gap" between any two reals because there is always a real there to fill that gap.

So we ask, where are all these gaps between reals you are talking about? Does a "gap" mean a place in the real line where there is no real point?

Re: Is continuum completely filled up?