Re: Is continuum completely filled up?

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- *From*: Dave Seaman <dseaman@xxxxxxxxxxxx>
- *Date*: Wed, 17 Jan 2007 23:59:25 +0000 (UTC)

On Wed, 17 Jan 2007 22:13:40 GMT, Andy Smith wrote:

> I
>
>> Distinct points on the line cannot have a separation of 0. If a and b are distinct real numbers, then | a − b | > 0.
>>
>> However, there is a way to represent the concept of things being "next to each other" on the line. That is to consider sets of points, rather than individual points. For example, let
>>
>> A = { x in Q : x < 0 } (the negative rationals),
>> B = { x in Q : x > 0 } (the positive rationals).
>>
>> The sets A and B are distinct, but they approach each other more closely than any positive real number. In fact, we can say that distance(A,B) = 0, where the distance between sets is defined by
>>
>> distance(X,Y) = inf{ |x−y| : x in X and y in Y }.
>>
>> Here, "inf" stands for "infimum", which means the greatest lower bound.
>>
>> But notice, even though distance(A,B) = 0, there is nevertheless a gap between the sets. The number 0 lies strictly between A and B and does not belong to either.
>>
>> The point is, requiring a distance to be zero is not sufficient to guarantee the absence of a gap. After all, points have zero width and therefore they can fit into a gap without causing any actual separation.
>>
>> This is why the notion of completeness does not rely on distance. Instead, completeness uses the least upper bound property. The set A does not have a maximum element, but it does have a least upper bound, and the LUB is a real number. The same is true of any nonempty set of reals that is bounded above.
>
> Thank you, understood that, and I can see that things are inevitably

> more tricky than they might seem. But I don't see that that answers my
> simplistic argument about gaps between the numbers?

I don't understand what it is that you think has not been answered. You seemed to be struggling for some way to reduce the distance between points to 0. I did three things: (1) I showed you how to reduce the distance to 0 by considering sets of points instead of individual points, (2) I pointed out that reducing the distance to zero is not sufficient to preclude gaps, and (3) I showed you how the concept of completeness addresses the problem of gaps without using distance.

If that isn't answering your question, then I evidently have not understood your question. Please ask it again.

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Dave Seaman
U.S. Court of Appeals to review three issues
concerning case of Mumia Abu−Jamal.
<http://www.mumia2000.org/>
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