Re: Is continuum completely filled up?

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- *From*: Andy Smith <Andy@xxxxxxxxxxxxxxxxxxxx>
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Dave Seaman writes

> Ok, so far.

Actually the iteration as specified gives 3^N after ~N iterations; I should have said to insert 1 real in each hole on each iteration, but this doesn't affect the line of argument.

My underlying train of thought was that you define a real via a Dedikind cut, which implicitly asserts a meaning to infinite sets. I was trying to consider a systematic method of creating all the reals from the bottom up (or maybe top down, depending on your perspective).

> > − if you have a countably infinite number of bits, then after a countably infinite number of iterations, you still have as many open intervals as distinct real numbers?
>
> That line of reasoning holds only as long as there is such a thing as a "next" point in the set, since an interval lies between a point and its immediate neighbor. There is not "next" point in the rationals, and therefore there are no intervals in the sense that you describe.

But at each level of the iteration you have an ordered set of 2^N points − you have defined them, so you know where they are? If one wished we could take e.g. pi or sqrt(2) and successively map that by proportionately scaling its location into each interval on each iteration, and we can then have a formula that describes the set of locations of all points at iteration N ?

The iterative construction puts "interval" (absence of specified points) on an equal footing with number, with a location defined (if you like) by the real number corresponding to its mid−point.

But doubtless you are right. As I said before, I don't trust any form of reasoning about infinities − you just have to consider
the Grandi series +1 −1 +1 −1 ... for which one can construct apparently solid arguments for the sum being any integer that you wish (of course, for series this would not be defined as convergent, but that rather dusts under the carpet the issue of why the logic is flawed − even if the series is non−convergent, it should be clear why a particular line of argument, e.g. add successive terms, is invalid, without circular reasoning (e.g. we

can't discuss this because it is a non−convergent series)).

> And, even though there are no intervals, there are still gaps in the rationals.

Well I think one could probably modify the argument in such a way as to demonstrate that there are still gaps in the rationals, but one doesn't need to do that because it is easy to show that there exists an irrational between any two rationals anyway.

> Properties of finite sets do not necessarily extend to infinite sets.

I can see that that is true, for sure.

Is there enough information in a countably infinite set of bits to encode every location on the real line? You would say yes, I guess.

−−
Andy Smith
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